

3.1

# UTILITY FUNCTION

USUALLY 1 EURO IS WORTH ALMOST NOTHING, BUT IN ANY CASE BETTER THAN 0 EURO.

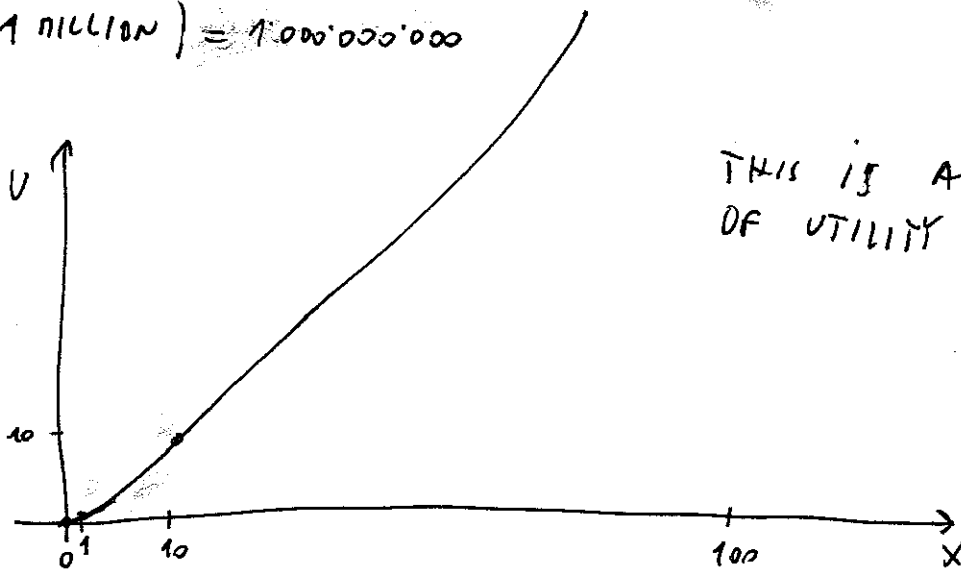
$$UTILITY(0) = 0 \quad UTILITY(1) = 0,001$$

10 EURO ARE USUALLY PERCEIVED AS THEIR RIGHT VALUE  
 $UTILITY(10) = 10$

100 EURO ARE CONSIDERED A NICE AMOUNT, 1000 EVEN BETTER  
 $UTILITY(100) = 120 \quad UTILITY(1000) = 2500$

10'000 EURO ARE WONDERFUL, AND 100'000 CAN SLIGHTLY CHANGE YOUR LIFESTYLE  
 $U(10'000) = 30000 \quad U(100'000) = 10'000'000$

ONE MILLION EURO CAN RADICALLY CHANGE YOUR LIFE  
 $U(1 \text{ MILLION}) = 1'000'000'000$



THIS IS AN EXAMPLE OF UTILITY FUNCTION

UTILITY FUNCTION HAS ONLY TWO REQUISITES

- 1)  $U(0) = 0$  (THIS REQUISITE IS OFTEN NOT CONSIDERED)
- 2)  $U$  IS STRICTLY INCREASING

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## ANOTHER EXAMPLE

1 MILLION CAN CHANGE YOUR LIFE  $U(1'000'000) = 1'000'000'000$   
 BUT 2 MILLIONS WILL NOT IMPROVE YOUR LIFE MUCH MORE

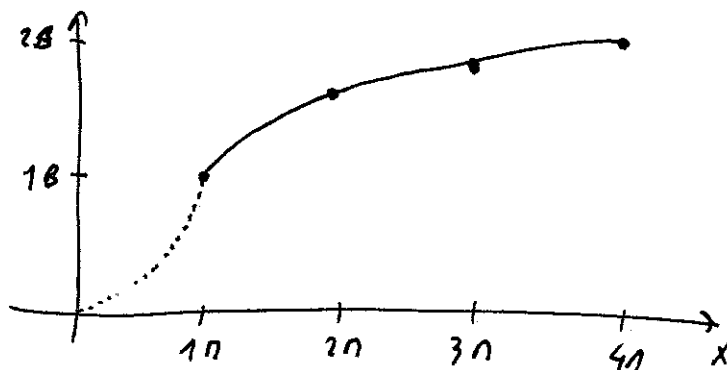
$$U(2'000'000) = 1'500'000'000$$

3 MILLIONS ARE BETTER, BUT NOT SO MUCH

$$U(3'000'000) = 1'700'000'000$$

4 MILLIONS DO NOT MAKE ANY CHANGE

$$U(4'000'000) = 1'800'000'000$$



UTILITY FUNCTION, WHICH MUST BE INCREASING, CAN THEREFORE BE CONVEX OR CONCAVE (OR STRAIGHT)

USING THE UTILITY FUNCTION, WE CAN EVALUATE THE EXPECTED UTILITY OF A LOTTERY

$$E(U(L)) = U(L) = \sum_{i=1}^s U(x_i) \cdot p_i$$

- SUM OF 2 DICE ROLLS WITH  $U(x) = e^x - 1$

$$E(U(L)) = U(L) = (e^{-1} - 1) \cdot \frac{2}{3} + (e^0 - 1) \cdot \frac{1}{6} + (e^4 - 1) \cdot \frac{1}{6} \approx -0,64 \cdot \frac{2}{3} + 0 + 53 \cdot \frac{1}{6} =$$

$$\approx 8,41$$

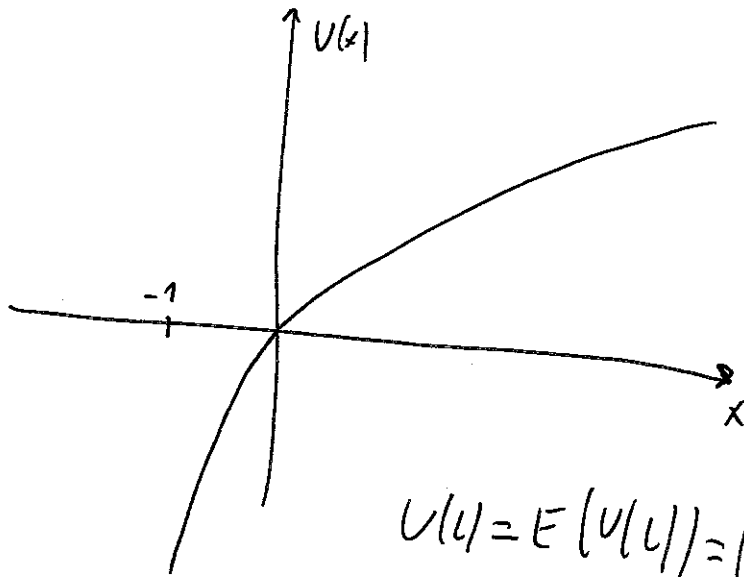


IF SOMEBODY HAS UTILITY  $e^x - 1 \Rightarrow$  SUM OF 2 DICE BECOMES CONVENIENT

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WE NOW CONSIDER

$$U(x) = \ln(x+e) - 1$$



$$L = (-1, \frac{2}{3}; 0, \frac{1}{6}; +4, \frac{1}{6})$$

$$\begin{aligned} U(L) = E[U(L)] &= (\ln(-1+e) - 1) \cdot \frac{2}{3} + (\ln(0+e) - 1) \cdot \frac{1}{6} + \\ &+ (\ln(4+e) - 1) \cdot \frac{1}{6} = -0,46 \cdot \frac{2}{3} + 0 \cdot \frac{1}{6} + 0,9 \cdot \frac{1}{6} = \\ &= -0,16 \end{aligned}$$

$$U(E(L)) = U(-1 \cdot \frac{2}{3} + 0 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6}) = U(0) = (\ln(0+e) - 1) = 0$$

LOTTERY FOR THIS USER IS NOT CONVARIANT SINCE  $U(L) = -0,16$

IN GENERAL WE WILL PROVE THAT CONCAVE FUNCTIONS BELONG TO RISK-AVERSE AGENTS AND CONVEX FUNCTIONS TO RISK-LOVER AGENTS