5.1 RISK-PREMIUM AND CERTAINTY-EQUIV.

DEF: RISK PREMIUM IS THE MAX AMOUNT OF MONEY THE AGENT WOULD PAY TO AVOID A PURE RISK LOTTERY

$$E(U(w_0+L)) = U(w_0 - P^r)$$

P^r DEPENDS ON w_0, U, AND ON L

IT IS DEFINED ONLY FOR PURE-RISK LOTTERIES

EXERCISE: $U(x) = e^x$ $\quad L = (1, 1/2; -1, 1/2)$

FIND RISK PREMIUM FOR ANY w_0

$E(L) = 0 \Rightarrow$ IT IS A PURE-RISK LOTTERY

$$E(U(w_0+L)) = U(w_0 - P^r)$$

$$\sum_{i=1}^{2} e^{w_0+x_i} \cdot p_i = e^{w_0 - P^r}$$

$$e^{w_0+1} \cdot \frac{1}{2} + e^{w_0-1} \cdot \frac{1}{2} = e^{w_0 - P_r}$$

$$\ln \frac{1}{2} e^{w_0} \left(e^1 + e^{-1} \right) = \ln e^{w_0 - P_r}$$

$$\ln e^{w_0} + \ln \frac{e+e^{-1}}{2} = w_0 - P_r$$

$$P^r = w_0 - w_0 - \ln \frac{e+e^{-1}}{2} = -\ln \frac{e+e^{-1}}{2} \simeq -0,431$$ IT DOES NOT DEPEND ON w_0

IT IS NEGATIVE BECAUSE U IS CONVEX $\Rightarrow$ AGENT IS RISK LOVER. $\Rightarrow$ HE WILL PAY TO PARTICIPATE IN THE LOTTERY AND NOT TO AVOID

5.2

HOMEWORK

EXERCISE: $U(x) = \ln x$;

FIND $\rho^\wedge$ FOR $L = (-1, 2/3; 0, 1/6; 4, 1/6)$

FOR ANY $W_0 > 1$

$$L = \left(-1, 2/3; 0, 1/6; 4, 1/6\right)$$

$E(L) = 0$ SO WE MAY CALCULATE $\rho^\wedge$

$$E(U(W_0 + L)) = U(W_0 - \rho^\wedge)$$

$$\left(\ln(W_0 - 1)\right)\cdot\frac{2}{3} + \left(\ln(W_0 + 0)\right)\cdot\frac{1}{6} + \left(\ln(W_0 + 4)\right)\cdot\frac{1}{6} = \ln(W_0 - \rho^\wedge)$$

$$\ln(W_0 - 1)\cdot\frac{2}{3} + \ln W_0 \cdot\frac{1}{6} + \ln(W_0 + 4)\cdot\frac{1}{6} = \ln(W_0 - \rho^\wedge)$$

APPLY EXP TO BOTH SIDES

$$\exp\left(\ln(W_0 - 1)\cdot\frac{2}{3} + \ln W_0 \cdot\frac{1}{6} + \ln(W_0 + 4)\cdot\frac{1}{6}\right) = W_0 - \rho^\wedge$$

$$\rho^\wedge = W_0 - \exp\left(\ln(W_0 - 1)\cdot\frac{2}{3} + \ln W_0 \cdot\frac{1}{6} + \ln(W_0 + 4)\cdot\frac{1}{6}\right) \quad \square$$

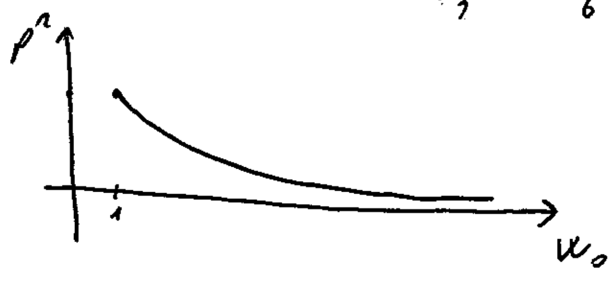
LET'S LOOK SOME VALUES OF $\rho^\wedge$

$W_0 = 2 \quad \rho^\wedge = 2 - \exp\left(\ln 1\cdot\frac{2}{3} + \ln 2\cdot\frac{1}{6} + \ln 6\cdot\frac{1}{6}\right) = 0{,}487$

$W_0 = 3 \quad \rho^\wedge = 3 - \exp\left(\ln 2\cdot\frac{2}{3} + \ln 3\cdot\frac{1}{6} + \ln 7\cdot\frac{1}{6}\right) = 0{,}36$

$W_0 = 4 \quad \rho^\wedge = 4 - \exp\left(\ln 3\cdot\frac{2}{3} + \ln 4\cdot\frac{1}{6} + \ln 8\cdot\frac{1}{6}\right) = 0{,}29$

WHEN $W_0 \to 1 \quad \rho^\wedge \to 1 - \exp\left(\ln 0\,\frac{2}{3} + \ln 1\cdot\frac{1}{6} + \ln 4\cdot\frac{1}{6}\right) = 1 - e^{-\infty} = 1$



$\rho^\wedge > 0$ SINCE AGENT IS STRICTLY RISK-AVERSE

5.3 $U = -e^{-x}$ $\quad L \sim N(0; \theta^2)$ FIND P^{π} $\quad w_0 = 0$

L IS PURE-RISK

$$E(U(L + w_0)) = U(w_0 - P^{\pi})$$

$$E(U(L)) = U(-P^{\pi})$$

$$E(U(L)) = \int_{-\infty}^{+\infty} -e^{-s} \cdot \frac{e^{-\frac{s^2}{2\theta^2}}}{\sqrt{2\pi\theta^2}} ds = -\int_{-\infty}^{+\infty} \frac{e^{-\frac{s^2 + 2\theta^2 s}{2\theta^2}}}{\sqrt{2\pi\theta^2}} ds$$

LET'S TRY TO HAVE A PERFECT SQUARE SUCH $(S + A)^2 =$

$= s^2 + 2sA + A^2$

WE ADD AND SUBTRACT θ^4

$$E(U(L)) = -\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\theta^2}} e^{-\frac{s^2 + 2\theta^2 s + \theta^4}{2\theta^2} - \frac{-\theta^4}{2\theta^2}} ds =$$

$$= -e^{\frac{\theta^2}{2}} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\theta^2}} e^{-\frac{(s+\theta^2)^2}{2\theta^2}} ds = -e^{\frac{\theta^2}{2}} \int_{-\infty}^{+\infty} f_{N(-\theta^2, \theta^2)}(s)\, ds =$$

$$= -\exp\left(\frac{\theta^2}{2}\right) \cdot 1$$

$$U(-P^{\pi}) = -e^{P^{\pi}} \qquad -e^{\frac{\theta^2}{2}} = -e^{+P^{\pi}} \implies \frac{\theta^2}{2} = P^{\pi}$$

$\square$

5-4) TAKE A GENERIC LOTTERY L

C^e <u>CERTAINTY EQUIVALENT</u>

IS A DETERMINISTIC LOTTERY $(C^e, 100\%)$ WHICH IS, FOR THE AGENT, EQUIVALENT TO L

$$E\left(U\left(w_0+L\right)\right)=U\left(w_0+C^e\right)$$

<u>NOTE</u>: RISK-PREMIUM IS DEFINED ONLY FOR PURE-RISK LOTTERIES

C^e IS DEFINED FOR EVERY LOTTERY

ALSO IN THIS CASE $C^e = C^e\left(w_0, L, U\right)$ DEPENDS ON w_0, L AND U

<u>IF</u> L is PURE-RISK $U\left(w_0-\rho^a\right)=E\left(U\left(w_0+L\right)\right)=U\left(w_0+C^e\right)$

WHEN U is STRICTLY INCREASING, WE CAN REMOVE IT AND

$$w_0-\rho^a=w_0+C^e \Longrightarrow \boxed{-\rho^a=C^e}$$ FOR L <u>PURE-RISK</u>

<u>IF</u> L IS GENERIC, $L-E(L)$ is PURE-RISK

$E\left(U\left(w_0+L\right)\right)=U\left(w_0+C^e\right)\ \underline{\forall w_0}$ $\quad E\left(U\left(w_0+L-E(L)\right)\right)=U\left(w_0'-\rho^a\right)\ \underline{\forall w_0'}$

$w_0'=w_0+E(L)\ \underline{\forall w_0}$

$\forall w_0\ E\left(U\left(w_0+L\right)\right)=U\left(w_0+C^e\right)$ $\quad E\left(U\left(w_0+L\right)\right)=U\left(w_0+E(L)-\rho^a\right)\ \underline{\forall w_0}$

$\Downarrow$ WHEN U is STRICTLY INCREASING

$$C^e\left(w_0; L; U\right)=E(L)-\rho^a\left(w_0+E(L); L-E(L); U\right)$$

<u>NOTE</u> THAT ρ^a IS THE RISK-PREMIUM OF $L-E(L)$ AND USES $w_0+E(L)$ (<u>NOT</u> OF L)

5.5 BASIC RULES FOR ρ^r AND C^e

AGENT IS RISK-AVERSE:

$\iff \rho^r > 0 \quad \forall L$ PURE-RISK $\underline{\underline{\forall w_0}}$

$\iff C^e < E(L) \quad \forall L$

AGENT IS RISK-LOVER:

$\iff \rho^r < 0 \quad \forall L$ PURE-RISK $\underline{\underline{\forall w_0}}$

$\iff C^e > E(L) \quad \forall L$

IF L HAS ONLY POS/NEG OUTCOMES, C^e IS POS/NEG $\forall$ AGENT

$\rho^r = -C^e$ IS VALID <u>ONLY</u> FOR L PURE-RISK

OTHERWISE THE RELATION IS NOT SO EASY AND INVOLVES $E(L)$

"CHEAPER" MEANS "SMALLER C^e"

AGENTS <u>ALWAYS</u> PREFER LOTTERIES WITH LARGER C^e

5.6 WHY ρ^{π} FOR A RISK-AVERSE AGENT IS NON-NEGATIVE?

DEFINITION OF ρ^{π}:

$$E(U(w_0+L)) = U(w_0-\rho^{\pi}) \quad \forall L \text{ WITH } E(L)=0$$

DEFINITION OF NON-STRICTLY RISK-AVERSE:

$$E(U(w_0+L)) \leq U(E(w_0+L)) \quad \forall w_0 \ \forall L$$

$$\Updownarrow$$

$$U(w_0-\rho^{\pi}) \leq U(E(w_0+L)) = U(w_0+E(L)) \quad \forall L \text{ PURE-RISK}$$

$$\Updownarrow$$

$$U(w_0-\rho^{\pi}) \leq U(w_0)$$

SINCE U IS INCREASING $\Updownarrow$

$$w_0-\rho^{\pi} \leq w_0$$

$$\Updownarrow$$

$$\rho^{\pi} \geq 0$$

$\square$

NOTE THAT SINCE ALL IMPLICATIONS ARE BIDIRECTIONAL, THIS ALSO SHOWS THAT IF $\rho^{\pi}(U, L, w_0) \geq 0 \ \forall L$ PURE-RISK AND $\forall w_0$ THEN THE AGENT IS NON-STRICTLY RISK-AVERSE

5.6B

THEOREM: A IS NOT LESS RISK-AVERSE THAN B

$\Updownarrow$

$\forall L$ $\quad C^e$ OF A IS $\leq$ C^e OF B CALCULATED FOR THE SAME w_0 AND FOR THE SAME L

$\Updownarrow$

$\forall w_0, \forall L$ PURE RISK, ρ^1 OF A IS $\geq$ ρ^1 OF B CALCULATED FOR THE SAME w_0 AND L

NOTE THAT C^e MUST HOLD "FOR EVERY L" BUT DOES NOT REQUIRE "FOR EVERY w_0"

EXERCISE: $U(x) = -e^{-x}$ $\quad L = (1, p; 0, 1-p)$

FIND C^e FOR ANY w_0

$$E(U(w_0+L)) = U(w_0+C^e)$$

$$-e^{-(w_0+1)}\cdot p + (-e^{-(w_0+0)})(1-p) = -e^{-(w_0+C^e)}$$

$$e^{-w_0-1}\cdot p + e^{-w_0}\cdot(1-p) = e^{-w_0-C^e}$$

$$\ln\left(e^{-w_0-1}\cdot p + e^{-w_0}(1-p)\right) = -w_0 - C^e$$

$$C^e = -w_0 - \ln\left(e^{-w_0-1}\cdot p + e^{-w_0}(1-p)\right)$$

$$C^e = -w_0 - \ln\left(e^{-w_0}\left[e^{-1}p + (1-p)\right]\right) = -w_0 - \ln\left(e^{-w_0}\right) - \ln\left(pe^{-1} + 1 - p\right) =$$

$$= -\cancel{w_0} + \cancel{w_0} - \ln\left(pe^{-1} + 1 - p\right) = -\ln\left(\frac{p}{e} + 1 - p\right)$$ IT DOES NOT DEPEND ON w_0

NOTE THAT $1 + \frac{p}{e} - p = 1 - p\left(1 - \frac{1}{e}\right) = 1 - p\left(\frac{e-1}{e}\right)$ IS BETWEEN e^{-1} AND 1

AND THEREFORE C^e IS BETWEEN 0 AND 1 $\Rightarrow C^e \geq 0$

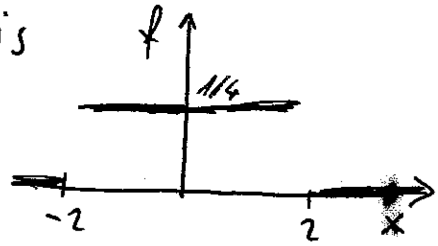
OBVIOUS SINCE L OUTCOMES ARE ALL ≥ 0 $\square$

5.7

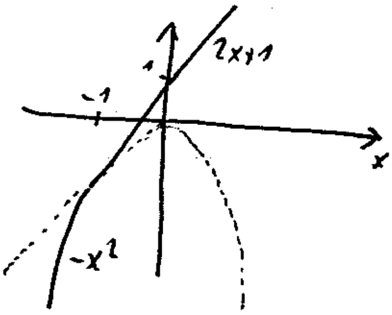
$$U(x) = \begin{cases} -x^2 & x \leq -1 \\ 2x+1 & x > -1 \end{cases} \qquad w_0 = 0 \qquad L \sim R(-2,2)$$

FIND C^e

$R(-2,2)$ is



$U(x)$ is



$$E(U(w_0+L)) = U(w_0 + C^e)$$

$$E(U(L)) = U(C^e)$$

$$E(U(L)) = \int_{-2}^{2} U(s) \cdot \frac{1}{4}\, ds = \int_{-2}^{-1} (-s^2)\, \frac{1}{4}\, ds + \int_{-1}^{2} (2s+1)\, \frac{1}{4}\, ds =$$

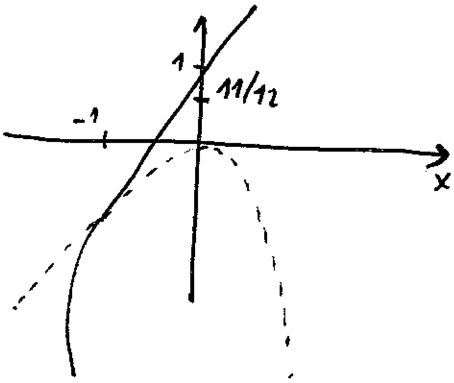
$$= \left[-\frac{s^3}{12}\right]_{-2}^{-1} + \left[\frac{s^2}{4} + \frac{s}{4}\right]_{-1}^{2} = +\frac{1}{12} - \frac{8}{12} + \frac{4}{4} + \frac{2}{4} - \frac{1}{4} + \frac{1}{4} =$$

$$= \frac{1-8+12+6-3+3}{12} = \frac{11}{12}$$

$$\frac{11}{12} = U(C^e)$$

FOR U, DO WE CHOOSE $-x^2$ OR $2x+1$?

LET'S LOOK AT THE GRAPH



A VALUE OF $\frac{11}{12}$ ON THE y AXIS CORRESPONDS TO THE ZONE WHERE U IS $2x+1$

$$\Downarrow$$

$$\frac{11}{12} = 2C^e + 1 \Rightarrow C^e = -\frac{1}{24}$$

IT IS NEGATIVE INFACT THE AGENT IS NON-STRICTLY RISK-AVERSE AND LOTTERY IS PURE-RISK

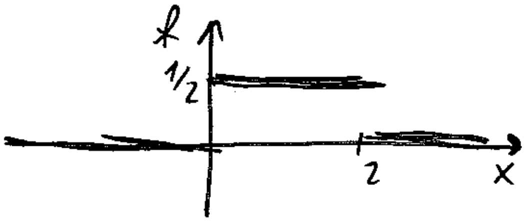
□

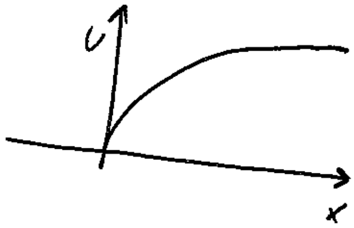
5.8

HOMEWORK

$U(x) = \sqrt{x} \quad x \geq 0 \qquad L \sim R(0,2)$

$w_0 = 2 \qquad$ FIND C^e

R(0,2) is 

U is 

$$E(U(w_0+L)) = U(w_0 + C^e)$$

$$E(U(2+L)) = U(2+C^e)$$

$$\int_0^2 \sqrt{s+2} \cdot \frac{1}{2}\, ds = \sqrt{2+C^e} \qquad \left[\frac{2}{3}(s+2)^{3/2} \cdot \frac{1}{2}\right]_0^2 = \sqrt{2+C^e}$$

$$\frac{(2+2)^{3/2}}{3} - \frac{(2+0)^{3/2}}{3} = \sqrt{2+C^e} \qquad \frac{\sqrt{4^3}}{3} - \frac{\sqrt{2^3}}{3} = \sqrt{2+C^e}$$

$$\frac{8}{3} - \frac{\sqrt{8}}{3} = \sqrt{2+C^e} \qquad 1{,}72 \approx \sqrt{2+C^e}$$

$$C^e \approx 0{,}97$$

IT IS POSITIVE. IN FACT, LOTTERY HAS ONLY POSITIVE OUTCOMES AND ITS $C^e > 0$ EVEN WHEN AGENT, AS IN THIS CASE, IS RISK-AVERSE

5.9 $U(x) = x^3$ $\quad L \sim LN(\mu, \theta^2)$ $\quad w_0 = 0$

FIND C^e

$$E(U(L+w_0)) = U(C^e + w_0)$$

$$E(U(L)) = U(C^e) = (C^e)^3$$

$$E(U(LN(\mu,\theta^2))) = \int_0^{+\infty} s^3 \cdot \frac{1}{s} \cdot \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(\ln s - \mu)^2}{2\theta^2}}\, ds =$$

$$= \int_0^{+\infty} e^{3\ln s}\, \frac{1}{s} \cdot \frac{1}{\sqrt{2\pi\theta^2}}\, e^{-\frac{(\ln s-\mu)^2}{2\theta^2}} = \int_0^{+\infty} \frac{1}{\sqrt{2\pi\theta^2}}\, \frac{1}{s}\, e^{-\frac{\ln^2 s + \mu^2 - 2\ln s\,\mu - 2\theta^2 \cdot 3\ln s}{2\theta^2}}\, ds =$$

NOW WE TRY TO HAVE A PERFECT SQUARE SUCH AS $(\ln s - A)^2$ ADDING AND SUBTRACTING THE SAME QUANTITY.

THE DOUBLE PRODUCT IS $-2\ln s(\mu + 3\theta^2)$ AND THE SQUARE IS $\ln^2 s$

THEREFORE THE TERM ~~m~~ MUST BE $(\mu + 3\theta^2)$ AND WE ADD $\pm(\mu+3\theta^2)^2$

$$= \int_0^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \frac{1}{s} \cdot e^{-\frac{\ln^2 s - 2\ln s(\mu+3\theta^2) \pm (\mu+3\theta^2)^2 + \mu^2}{2\theta^2}}\, ds =$$

$$= \underbrace{\int_0^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \frac{1}{s} \cdot e^{-\frac{(\ln s - (\mu+3\theta^2))^2}{2\theta^2}}}_{} \cdot e^{-\frac{-(\mu+3\theta^2)^2 + \mu^2}{2\theta^2}}\, ds =$$

THIS IS $\int_0^{+\infty} f_{LN(\mu+3\theta^2;\, \theta^2)}\, ds$ WHICH IS 1

5.10

$$= 1 \cdot e^{\frac{\mu^2+9\sigma^4+6\mu\sigma^2-\mu^2}{2\sigma^2}} = e^{\frac{9\sigma^2+6\mu}{2}}$$

$$C^\ell = \sqrt[3]{e^{\frac{9\sigma^2+6\mu}{2}}} = e^{\frac{9\sigma^2+6\mu}{6}} = e^{\frac{3\sigma^2+2\mu}{2}} = e^{\mu+\frac{3}{2}\sigma^2}$$

$\square$

5.11)

HOMEWORK $U(x) = x^{1.5}$ $x \in (0, +\infty)$ UTILITY

WHAT ARE RISK PREFERENCES OF THIS AGENT ?

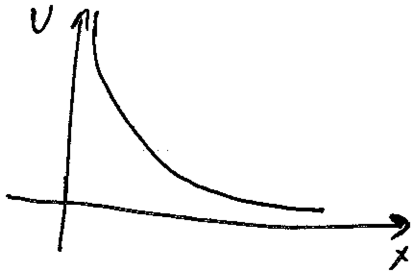
FIND C^e FOR $L \sim LN(\mu, \sigma^2)$ WITH $w_0 = 0$

$$U' = 1.5\, x^{0.5} = 1.5 \sqrt{x} = \frac{3}{2}\sqrt{x}$$

$$U'' = \frac{3}{4\sqrt{x}} \qquad x \in (0, +\infty) \qquad U'' > 0 \text{ IN } (0, +\infty)$$

$\Downarrow$

AGENT IS RISK - LOVER



$$E(U(L + w_0)) = U(w_0 + C^e) \qquad E(U(LN(\mu, \sigma^2))) = U(C^e)$$

$$E(U(LN(\mu, \sigma^2))) = \int_0^{+\infty} s^{1.5} \frac{1}{s} \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(\ln s - \mu)^2}{2\sigma^2}}\, ds$$

$$s^{1.5} = e^{\ln s^{1.5}} = e^{1.5 \ln s} \qquad (\ln s - \mu)^2 = +\ln^2 s + \mu^2 - 2\mu \ln s$$

$$E(U(LN)) = \int_0^{+\infty} \frac{1}{s} \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{\ln^2 s - 2(1.5\sigma^2 + \mu)\ln s + \mu^2}{2\sigma^2}}\, ds$$

I WANT TO GET A PERFECT SQUARE IN THE FORM $(\ln s - A)^2 =$
$= \ln^2 s - 2 \cdot A \cdot \ln s + A^2$

THEREFORE I ADD AND SUBTRACT $(1.5\sigma^2 + \mu)^2$

5.12)

$$E(U(LN)) = \int_0^{+\infty} \frac{1}{s}\frac{1}{\sqrt{2\pi\theta^2}} e^{-\frac{\ln^2 s - 2(1.5+\mu)\ln s + (1{,}5\theta^2+\mu)^2}{2\theta^2} - \frac{\mu^2 - (1{,}5\theta^2+\mu)^2}{2\theta^2}} ds$$

$$= e^{-\frac{\mu^2 - (1{,}5\theta^2+\mu)^2}{2\theta^2}} \int_0^{+\infty} \frac{1}{s}\frac{1}{\sqrt{2\pi\theta^2}} e^{-\frac{(\ln s - (1{,}5\theta^2+\mu))^2}{2\theta^2}} ds =$$

$$\downarrow$$

$$= \exp\left(-\cancel{\frac{\mu^2}{2\theta^2}} + \frac{1.5^2\theta^4}{2\theta^2} + \cancel{\frac{\mu^2}{2\theta^2}} + \cancel{\frac{2\cdot 1{,}5\mu\theta^2}{2\theta^2}}\right) \int_0^{+\infty} f_{LN(1.5\theta^2+\mu;\theta^2)}(s)\, ds =$$

$$= \exp\left(\frac{1{,}5^2\theta^2}{2} + 1{,}5\mu\right) \cdot 1$$

$$U(C^e) = (C^e)^{1.5} \qquad (C^e)^{1.5} = e^{\theta^2\frac{1{,}5^2}{2} + 1{,}5\mu}$$

$$C^e = \exp\left(\theta^2 \frac{1{,}5}{2} + \mu\right)$$

$$\square$$

5.13) $W_0 = 0 \quad V = x^2 \quad x \geq 0$

$L_1 \sim R(0,2) \qquad L_2 \sim \lambda e^{-\lambda x} \qquad \underline{\lambda = 1}$

WHICH LOTTERY DOES AGENT PREFER?

WE CALCULATE CERTAINTY EQUIVALENT FOR L_1 AND L_2

LET'S USE THE C^e

$$E(U(L_1)) = \int_0^2 s^2 \frac{1}{2}\, ds = \left[\frac{s^3}{6}\right]_0^2 = \frac{8}{6} = \frac{4}{3}$$

$$U(C_1^e) = (C_1^e)^2$$

$$(C_1^e)^2 = \frac{4}{3} \qquad C^e = \pm\frac{2}{\sqrt{3}}$$

WE OBVIOUSLY CHOOSE + SINCE X OUTCOMES ARE ALL POSITIVE!

$$C_1^e = \frac{2}{\sqrt{3}}$$

$$E(U(L_2)) = \int_0^{+\infty} s^2 e^{-s}\, ds \underset{\text{PARTS}}{=} \left[-s^2 e^{-s}\right]_0^{+\infty} - \int_0^{+\infty} -2s\, e^{-s}\, ds \underset{\text{PARTS}}{=}$$

$$= -0 + 0 + 2\left[-s e^{-s}\right]_0^{+\infty} - 2\int_0^{+\infty} -1 \cdot e^{-s}\, ds = 0 + 2(-0+0) + 2\left[-e^{-s}\right]_0^{+\infty} =$$

$$= 0 + 2(-0+1) = +2 \qquad (C_2^e)^2 = 2 \qquad C_2^e = \sqrt{2}$$

$$C_2^e > C_1^e \Rightarrow \text{AGENT PREFERS LOTTERY } 2$$

$\square$

5.14

$W_0 = 0 \qquad U = x^3 \qquad \xi_1 \sim R(0;1) \qquad \xi_2 \sim LN(1;2)$

- HOMEWORK

WHICH LOTTERY DOES AGENT PREFER?

WE CALCULATE CERTAINTY EQUIVALENT

$$U(C_1^e) = E(U(\xi_1))$$

$$(C_1^e)^3 = \int_0^1 s^3 \cdot 1 \, ds = \left[\frac{s^4}{4}\right]_0^1 = \frac{1}{4} \qquad C_1^e = \frac{1}{\sqrt[3]{4}} \approx 0{,}63$$

$$U(C_2^e) = E(U(\xi_2))$$

$$(C_2^e)^3 = \int_0^{+\infty} s^3 \cdot e^{-\frac{(\ln s - 1)^2}{4}} \cdot \frac{1}{s} \frac{1}{\sqrt{2\pi \cdot 2}} \, ds =$$

WE USE THE TYPICAL TECHNIQUE FOR SOLVING $\int s^n f_{LN}(s) \, ds$

$$s^3 = \exp(\ln s^3) = e^{3 \ln s}$$

$$= \int_0^{+\infty} e^{3\ln s} \cdot e^{-\frac{\ln^2 s + 1 - 2\ln s}{4}} \cdot \frac{1}{s} \cdot \frac{1}{\sqrt{2\pi \cdot 2}} \, ds = \int_0^{+\infty} e^{-\frac{-12\ln s + \ln^2 s + 1 - 2\ln s}{4}} \cdot \frac{1}{s} \cdot \frac{1}{\sqrt{2\pi \cdot 2}} \, ds =$$

FROM $-14\ln s + \ln^2 s + 1$ WE TRY TO GET A PERFECT SQUARE

$$(\ln s - 7)^2 = \ln^2 s - 14 \ln s + 49$$

$$(C_2^e)^3 = \int_0^{+\infty} e^{-\frac{(\ln s - 7)^2 - 48}{4}} \frac{1}{s} \cdot \frac{1}{\sqrt{2\pi 2}} \, ds = e^{12} \int_0^{+\infty} e^{-\frac{(\ln s - 7)^2}{4}} \cdot \frac{1}{s} \cdot \frac{1}{\sqrt{2\pi 2}} \, ds =$$

$\boxed{5.15}$

$$= e^{12} \int_0^{+\infty} f_{LN(7;2)}(s)\, ds = e^{12} \cdot 1 = e^{12}$$

$$C_2^e = e^{\frac{12}{3}} = e^{4} \approx 53,9$$

THE AGENT WILL PREFER LOTTERY $\wp_2$

$\square$

5.16 $L \sim N(0;0{,}01)$ $U = -e^{-x}$ $w_0 = 1$

FIND C^e

$$E(U(L+w_0)) = U(w_0 + C^e)$$

$$E(U(L+w_0)) = \int_{-\infty}^{+\infty} -e^{-(s+1)} \cdot \frac{1}{\sqrt{2\pi 0{,}01}} \cdot e^{-\frac{(s-0)^2}{2\cdot 0{,}01}} ds =$$

$$= -\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi 0{,}01}} \cdot e^{-\frac{0{,}02\cdot(s+1)+s^2}{2\cdot 0{,}01}} ds =$$

$$= -\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi 0{,}01}} \cdot e^{-\frac{s^2+0{,}02\,s+0{,}02}{2\cdot 0{,}01}} ds$$

WE WANT A PERFECT SQUARE $(s+0{,}01)^2 = s^2+0{,}02s+0{,}01^2$

$$= -\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi 0{,}01}} \quad e^{-\frac{(s+0{,}01)^2-0{,}01^2+0{,}02}{2\cdot 0{,}01}} ds =$$

$$= -\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi 0{,}01}} \quad e^{-\frac{(s+0{,}01)^2}{2\cdot 0{,}01}} \cdot e^{-\frac{-0{,}01^2+0{,}02}{2\cdot 0{,}01}} ds =$$

$$= -e^{-0{,}995} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi 0{,}01}} \quad e^{-\frac{(s+0{,}01)^2}{2\cdot 0{,}01}} ds =$$

$$= -e^{-0{,}995} \int_{-\infty}^{+\infty} f_{N(0{,}01;0{,}01)}(s)\, ds = e^{-0{,}995}$$

5.17

$U(w_0 + C^e) = -e^{-(1+C^e)} = -e^{-1-C^e}$

$-e^{-0{,}995} = -e^{-1-C^e}$ $\qquad$ $e^{-0{,}995} = e^{-1-C^e}$

$-0{,}995 = -1 - C^e$

$\therefore C^e = -0{,}005$

C^e IS NEGATIVE : OK SINCE AGENT IS RISK AVERSE AND L IS PURE RISK

D

5.18

$U(x) = \exp(x)$

FIND $A(x)$ FIND P^r FOR $U(a;b)$ WITH w_0

CHECKING WHETHER L IS PURE-RISK

$U'(x) = \exp(x)$ $U''(x) = \exp(x)$ $A(x) = -1$

TO FIND P^r WE MUST ASSUME THAT THE CONTRACT IS PURE-RISK $\Rightarrow E(U(a,b)) = 0$

$\Rightarrow \frac{a+b}{2} = 0 \Rightarrow a = -b$

SO NOW $L \sim U(-b;b)$ (IN CASE $b<0$, WE CAN REDO THE SAME CALCULATIONS WITH $U(b;-b)$)

$E(U(w_0+L)) = U(w_0 - P^r)$ $U(w_0-P^r) = e^{w_0-P^r} = e^{w_0} \cdot e^{-P^r}$

$$E(U(w_0+L)) = \int_{-b}^{+b} e^{w_0+s} \cdot \frac{1}{2b}\, ds = \frac{e^{w_0}}{2b} \cdot \int_{-b}^{b} e^s\, ds = \frac{e^{w_0}}{2b}\left[e^s\right]_{-b}^{b} =$$

$$= \frac{e^{w_0}\left(e^b - e^{-b}\right)}{2b}$$ $$\frac{e^b - e^{-b}}{2b} = e^{-P^r}$$ $$-P^r = \ln \frac{e^b - e^{-b}}{2b}$$

$$P^r = -\ln \frac{e^b - e^{-b}}{2b}$$

$\square$