14.1 INVENTORY MANAGEMENT

PROBLEM: DECIDE HOW MANY PRODUCTS TO BUY.
FOR EVERY UNSOLD PRODUCT THERE IS AN
OVERAGE COST C_o, FOR EVERY NOT-SOLD PRODUCT
THERE IS AN UNDERAGE COST C_u

THE LOSS IS $\ell(x, \xi) = \max\left(C_u(\xi - x), C_o(x-\xi)\right)$

$L(x) = E(\ell(x,\xi))$ IS A CONVEX FUNCTION SINCE
THE LINEAR COMBINATION OF THE MAXIMUM OF LINEAR FUNCTIONS

$$L(x) = \int_0^{+\infty} \ell(x,s) f(s)\, ds = C_u \int_x^{+\infty} (s-x) f(s)\,ds + C_o \int_0^x (x-s) f(s)\, ds =$$

$$= C_u \int_x^{+\infty} s f(s)\,ds - C_u x \int_x^{+\infty} f(s)\,ds + C_o x \int_0^x f(s)\,ds - C_o \int_0^x s f(s)\,ds =$$

$$= C_u\left(E(\xi) - \int_0^x s f(s)\,ds\right) - C_u x\left(1-F(x)\right) + C_o x\, F(x) - C_o \int_0^x s f(s)\,ds.$$

$$L'(x) = \cancel{-C_u x f(x)} - C_u(1-F(x)) \cancel{+ C_u x f(x)} + C_o F(x) \cancel{+ C_o x f(x)} \cancel{- C_o x f(x)} =$$

$$= C_u F(x) - C_u + C_o F(x) = -C_u + F(x)(C_u + C_o)$$

A STATIONARY POINT IS $F(x^*) = \dfrac{C_u}{C_u + C_o}$ $\quad x^* = F^{-1}\left(\dfrac{C_u}{C_u+C_o}\right)$

x^* IS A MINIMUM OF THE EXPECTED LOSS
SINCE $L(x)$ IS CONVEX

14.2

EXERCISE $\xi \sim \lambda e^{-\lambda x}$

FIND x^* AS FUNCTION OF λ, c_u, c_o

$$F(x) = \int_0^x \lambda e^{-\lambda s} ds = \left[-e^{-\lambda s}\right]_0^x = 1 - e^{-\lambda x}$$

$$1 - e^{-\lambda x^*} = \frac{c_u}{c_u + c_o}$$

$$-\lambda x^* = \ln\left(1 - \frac{c_u}{c_u + c_o}\right) \qquad x^* = -\frac{1}{\lambda} \ln\left(1 - \frac{c_u}{c_u + c_o}\right)$$

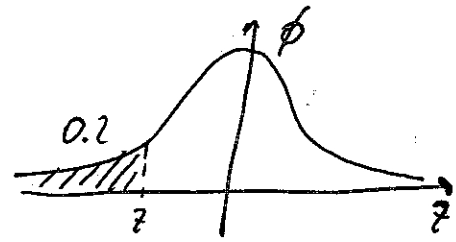
EXERCISE $\xi \sim LN(\mu, \sigma)$

FIND x^* WHEN $c_u = 0{,}5$ $c_o = 2$ $\mu = 7$ $\sigma^2 = 3$

$$F_{LN}(x^*) = \frac{c_u}{c_o + c_u} = \frac{0.5}{2.5} = 0.2$$

$$\phi\left(\frac{\ln x^* - 7}{\sqrt{3}}\right) = 0.2$$

LOOKING IN THE TABLES



$$z = -0.84$$

$$\frac{\ln x^* - 7}{\sqrt{3}} = -0.84 \qquad \ln x^* = 7 - 0{,}84 \cdot \sqrt{3} = 5.545$$

$$x^* = e^{5.545} = 251{,}68$$

RESULT IS NOT INTEGER BECAUSE WE USED A CONT. DISTR.

$\square$

14.3

NEWS BOY PROBLEM

$C_U = 4 \qquad C_O = 1$

DEMAND IS $R(0, \theta) \qquad \theta > 0$

FIND NUMBER OF COPIES THAT MINIMIZES EXPECTED LOSS

FUNCTION TO MINIMIZE IS

$$L(x) = E(\ell(x, \varphi))$$

$$L'(x) = -C_U + F(x)(C_U + C_O) = -4 + F(x)5$$

IN THIS CASE $F(x) = \int_0^x \frac{1}{\theta} ds = \frac{x}{\theta}$ WHEN $x \in [0, \theta]$

$F(x) = 0 \quad x < 0 \qquad F(x) = 1 \quad x > \theta$

$-4 + \frac{x^*}{\theta} 5 = 0$ TO FIND STATIONARY POINT ($x^* \in [0, \theta]$)

$\frac{x^*}{\theta} = \frac{4}{5} \qquad x^* = \frac{4\theta}{5}$ IS STATIONARY POINT.

SINCE x^* IS BETWEEN 0 AND θ, WE ARE OK.

$\square$