Tickets to the state lottery cost \$1. There are two possible prizes: a \$10 payoff with probability 1/50 and a \$1000000 payoff with probability 1/2000000. What is the expected monetary value of a lottery ticket? When (if ever) is it rational to buy a ticket? Be precise - show an equation involving utilities. You may assume that u(\$10) = 10 × u(\$1), but you may not make any assumptions about u(\$1000000). Sociological studies show that people with lower income buy a disproportionate number of lottery tickets. Do you think this is because they are worse decision makers or because they have a different utility function?

Consider the utility function $u(x)=a+bx+cx^2$. What assumptions have to be made on this function for it to represent risk averse agent who derives utility from x? Is the function valid for any value of x?

A farmer has at his disposal 100 acres of land. The land may be used for growing wheat or barley. The wheat yield depends on the weather (i.e. occurrence of frost), while the barley yield does not. Consequently, if the weather is good the farmer receives a yield on wheat land worth EUR 4500 per acre, whereas if the weather is bad the value of the wheat crop is EUR 1500 per acre only. The probability of bad weather is 1/3. The yield on barley is EUR 2500 in any case. Irrespective of the choice of crop, the cost of fertilisers, seeds etcetera is EUR 1000 per acre.

A. Assume the farmer choose to produce wheat only. Show that in this case his expected net income equals EUR 250000.

B. If the farmer only considers the two extremes, 'all barley' and 'all wheat', should he then choose all wheat? Explain!

C. Assume the farmer has preferences that satisfy the Expected Utility Theorem and that his utility function is u(x), where x is net income. What are the expressions for the farmer's expected utility in the two cases considered in B?

D. Assume now that the farmer may choose a mix between wheat and barley. Let Y denote that amount of land used for barley. Net income in good weather conditions is denoted x_G while net income in bad weather conditions is x_B. Show that

$x_G=350000 – 2000\ Y$

$x_B=50000 – 1000Y$

E. Derive the expression for the farmer's expected utility and show how it depends on Y. Explain how we may (in principle) find the optimal value of Y.

We consider a firm that may invest in a project with a risky outcome. If the firm invests an amount K, with probability p revenues are $R_H(K)$ and with probability $1-p$ revenues are $R_L(K)$, where $R_H(K) > R_L(K)$ and both are twice continuously differentiable, increasing and concave.

In the first part of the exercise, we assume that the unit cost of investment is $1+r$, where r is the (given) interest rate.

a. Consider the case in which the firm acts as if it does not care about risk. What is the expression for expected profits? Derive the first-order condition for the profit-maximising investment level. Interpret the result.
b. Consider next the case in which the firm acts as if it is risk averse. Can you give reasons for this assumption? Assume that the firm acts as if it evaluates profits according to the utility function u. What is the expression for expected utility of the firm? Derive the first-order condition for the expected-utility maximising investment level. Interpret the result. How does the resulting investment level compare to that under A?

Suppose that the firm has access to two different sources of finance. In addition to being able to borrow at the fixed interest rate r, the firm may also borrow at a risky rate. For simplicity we assume that this interest rate takes on the value r_H when revenues are R_H and the value r_L when revenues are R_L , where $r_H > r_L$.

c. Consider again the case of risk neutrality. Is it possible that the firm may choose to borrow from both sources? Under what conditions on the interest rates will the firm choose to borrow at the fixed and risky interest rate, respectively? Does the firm's choice depend on the riskiness of revenues?
d. Consider next the case of risk aversion. Explain why the firm would never borrow at the fixed interest rate if $r > r_H$. Explain also why the firm would never borrow at the risky interest rate if $r < r_L$. Suppose for the rest of the analysis that $r_L<r<r_H$. Is it conceivable that the firm will borrow at the risky interest rate even if $p\, r_H+(1-p)\, r_L>r$? Does the choice depend on the riskiness of revenues? Explain.

We return again to the case in which the firm can borrow at a fixed rate of interest only. However, we now assume that the profit of the firm is subject to a (proportional) tax t.

e. In the case of risk neutrality, does the rate of taxation affect investment behaviour? Explain.
f. What about in the case of risk aversion? Explain.

If you are exposed to a 50/50 probability of gaining or losing EUR 1000 and an insurance that removes the risk costs EUR 500, at what level of wealth will you be indifferent between taking the gamble or paying the insurance? That is, what is your certainty equivalent wealth for this gamble? Assume that your utility function is $u(Y) = -1/Y$.
What would the solution be if the utility function were logarithmic?

Assume that you have a logarithmic utility function on wealth $u(Y)=\ln Y$ and that you are faced with a 50/50 probability of winning or losing EUR 1000. How much will you pay to avoid this risk if your current level of wealth is EUR 10000? How much would you pay if your level of wealth is EUR 1000000? Did you expect that the premium you were willing to pay would increase/decrease? Why?

An agent faces a risky situation in which he can lose some amount of money with probabilities given in the following table:

Loss	Probability
1000	10%
2000	20%
3000	35%
5000	20%
6000	15%

This agent has a utility function of wealth of the form $u(Y) = \frac{Y^{1-\gamma}}{1-\gamma} + 2$

His initial wealth level is 10000 and his γ is equal to 1.2.

g. Calculate the certainty equivalent of this prospect for this agent. Calculate the risk premium. What would be the certainty equivalent of this agent if he would be risk neutral?
h. Describe the risk premium of an agent whose utility function of wealth has the form implied by the following properties: $u'(Y)>0$ and $u''(Y) > 0$

There is an individual with a well-behaved utility function, and initial wealth Y. Let a lottery offer a payoff of G with probability p and a payoff of B with probability 1-p.

a. a. If the individual already owns this lottery denote the minimum price he would sell it for by Ps. Write down the expression Ps has to satisfy.
b. b. If he does not own it, write down the expression Pb (the maximum price he would be willing to pay for it) has to satisfy.
c. c. Assume now that p=1/2, Y=10, G=6, B=26, and the utility function is u(Y)=Y½ . Find buying and selling prices. Are they equal? Explain why not. Generally, can they ever be equal?

Consider the following investments:

Investment 1		Investment 2	
Payoff	Prob.	Payoff	Prob.
1	0.25	3	0.33
7	0.50	5	0.33
12	0.25	8	0.34

Check that neither investment dominates the other on the basis of

- The Mean-Variance criterion
- First Order Stochastic Dominance
- Second Order Stochastic Dominance

How could you rank these investments?

Given the following probability distributions for risky payoffs x and z:

x	Probability(x)	z	Probability(z)
-10	.1	2	.2
5	.4	3	.5
10	.3	4	.2
12	.2	30	.1

a. If the only available choice is 100% of your wealth in x or 100% in z and you choose on the basis of mean and variance, which asset is preferred?
b. According to the second-order stochastic dominance criterion, how would you compare them?
