2.1 A $\xi = \begin{cases} 1 & p \\ 0 & 1-p \end{cases}$ BERNOULLI RANDOM VARIABLE

INDEPENDENT

$\varepsilon = \begin{cases} 1 & q \\ 0 & 1-q \end{cases}$ BERNOULLI RANDOM VARIABLE

CONSIDER $\xi \cdot \varepsilon$ $\max(\xi, \varepsilon)$ $\overset{HW}{\min(\xi, \varepsilon)}$

$\overset{HW}{\max(\xi, \varepsilon) - \min(\xi, \varepsilon)}$

DRAW VENN DIAGRAMS AND CHARACTERIZE THESE RANDOM VARIABLES

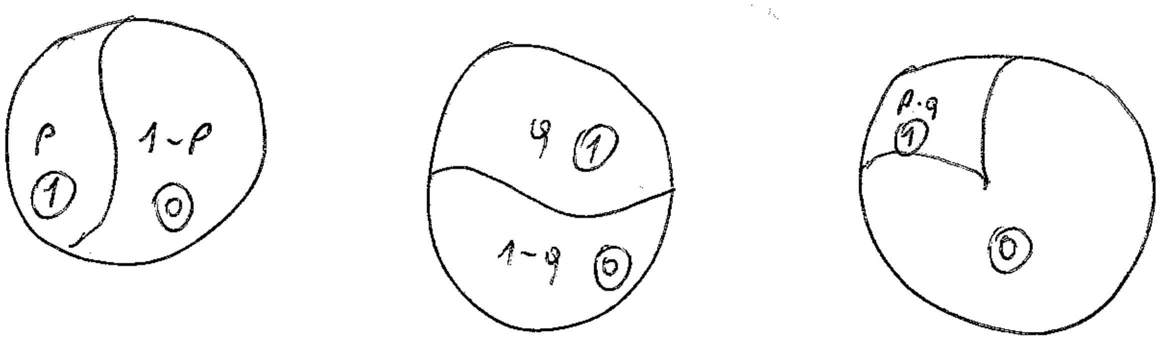
$\xi \cdot \varepsilon = \begin{cases} 1 & \text{WHEN BOTH } \xi \text{ AND } \varepsilon \text{ ARE } 1 \\ 0 & \text{WHEN AT LEAST ONE BETWEEN } \xi \text{ AND } \varepsilon \text{ IS } 0 \end{cases}$

$\Downarrow$

$\xi \cdot \varepsilon = \begin{cases} 1 & p \cdot q \\ 0 & (1-p)q + p(1-q) + (1-p)(1-q) \end{cases}$

$\Downarrow$

$\xi \cdot \varepsilon = \begin{cases} 1 & p \cdot q \\ 0 & 1 - p \cdot q \end{cases}$ IT IS A BERNOULLI VARIABLE WITH PROBABILITY $p \cdot q$



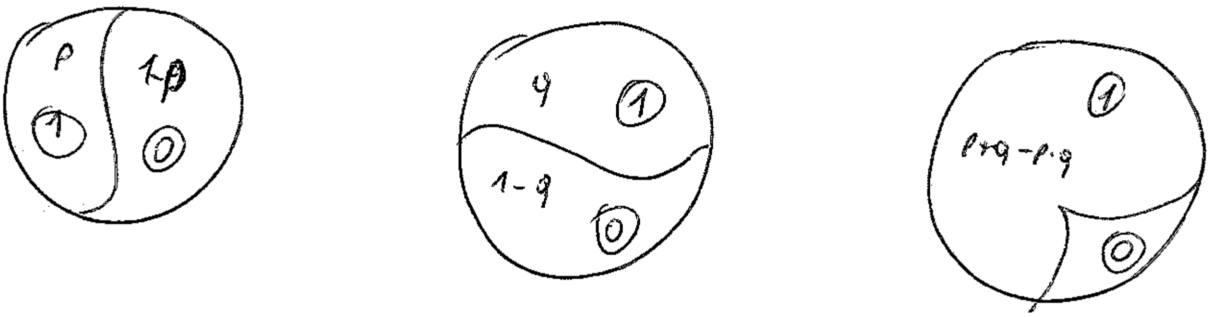
2.1 B

$$\max(\xi, \varepsilon) = \begin{cases} 1 & \text{WHEN AT LEAST ONE IS } 1 \\ 0 & \text{WHEN BOTH ARE } 0 \end{cases}$$

$$\Downarrow$$

$$\max(\xi, \varepsilon) = \begin{cases} 1 & p \cdot q + p(1-q) + (1-p) \cdot q = p + q - p \cdot q \\ 0 & (1-p)(1-q) = 1 - (p + q - p \cdot q) \end{cases}$$

IT IS A BERNOULLI RANDOM VARIABLE WITH PROBABILITY $p + q - p \cdot q$



$$\min(\xi, \varepsilon) = \begin{cases} 1 & \text{WHEN BOTH ARE } 1 \quad pq \\ 0 & \text{WHEN AT LEAST ONE IS } 0 \quad 1 - pq \end{cases}$$

IT IS EXACTLY LIKE $\xi \cdot \varepsilon$

$$\max(\xi, \varepsilon) - \min(\xi, \varepsilon) = \begin{cases} 1 - 1 & \text{WHEN } \xi \text{ AND } \varepsilon \text{ ARE } 1 \\ 1 - 0 & \text{WHEN ONE IS 1 AND THE OTHER IS } 0 \\ 0 - 1 & \text{NEVER} \\ 0 - 0 & \text{WHEN BOTH ARE } 0 \end{cases}$$

2.1 c

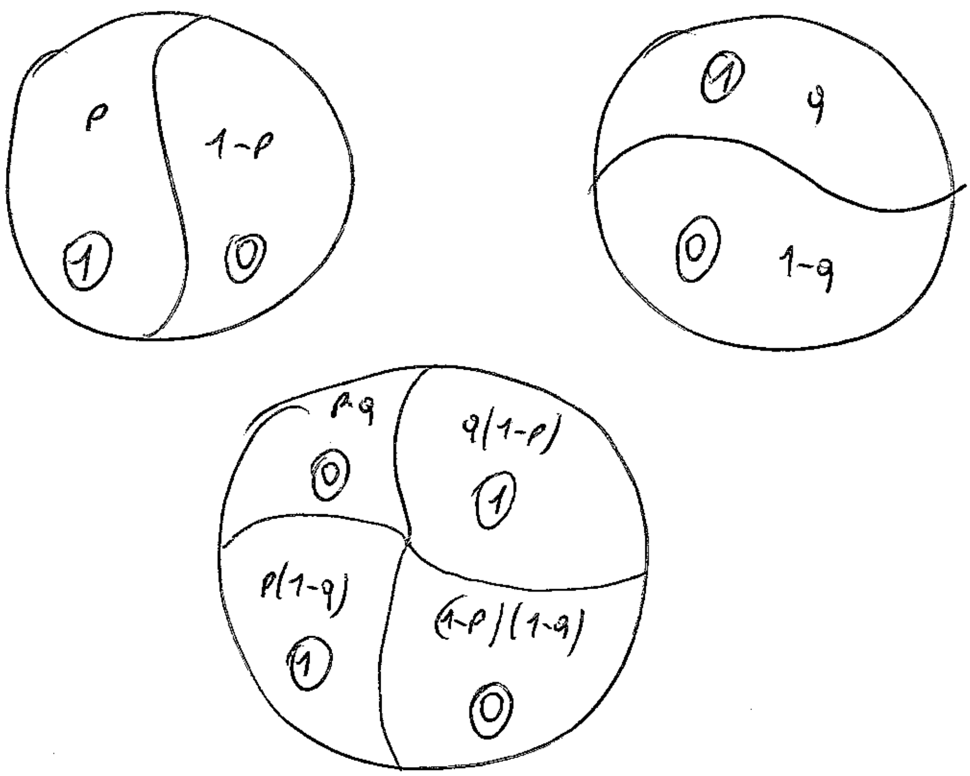
$$\max(\xi, \varepsilon) - \min(\xi, \varepsilon) = \begin{cases} 0 & \text{WHEN } \xi \text{ AND } \varepsilon \text{ ARE 1 OR 0} \\ 1 & \text{WHEN ONE IS 1 AND THE OTHER 0} \end{cases}$$

$$= \begin{cases} 0 & \text{WHEN } \xi \text{ AND } \varepsilon \text{ ARE EQUAL} \\ 1 & \text{WHEN } \xi \text{ AND } \varepsilon \text{ ARE DIFFERENT} \end{cases}$$

$$= \begin{cases} 0 & p \cdot q + (1-p)(1-q) = p \cdot q + 1 + pq - p - q \\ 1 & p(1-q) + q(1-p) = p - qp + q - qp \end{cases}$$

$$= \begin{cases} 0 & 1 - (p+q-2qp) \\ 1 & p+q-2qp \end{cases}$$

IT IS A BERNOULLI VARIABLE WITH PROBABILITY $p+q-2qp$

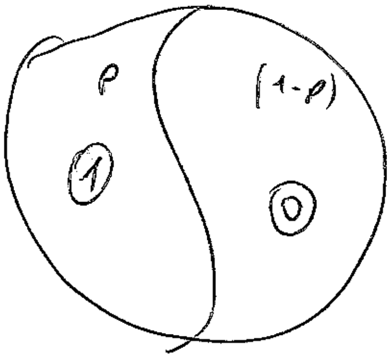


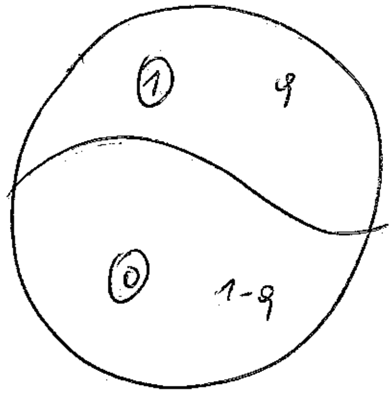
(2.2) A

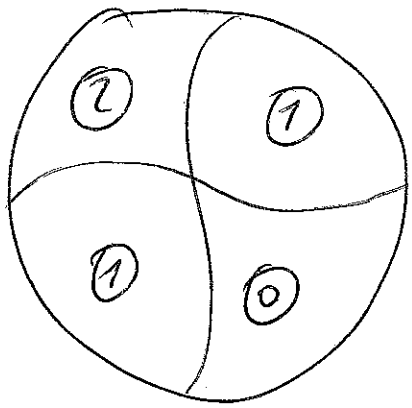
$$\xi = \begin{cases} 1 & p \\ 0 & 1-p \end{cases} \qquad \varepsilon = \begin{cases} 1 & q \\ 0 & 1-q \end{cases}$$

CHARACTERIZE $\quad \xi + \varepsilon \quad \overset{HW}{\xi - \varepsilon} \quad \overset{HW}{p^{\xi}}$

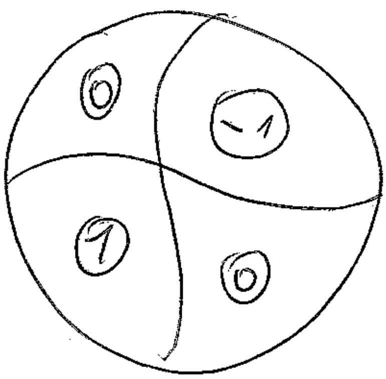
$$\xi + \varepsilon = \begin{cases} 1+1 & pq \\ 1+0 & p(1-q) \\ 0+1 & (1-p)q \\ 0+0 & (1-p)(1-q) \end{cases} = \begin{cases} 2 & pq \\ 1 & -2pq+p+q \\ 0 & (1-p)(1-q) \end{cases}$$





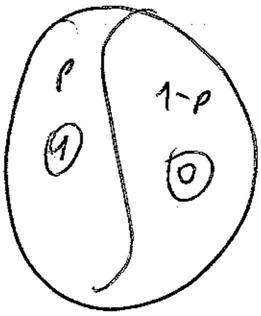


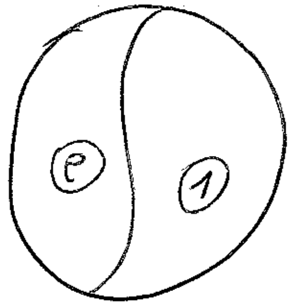
$$\xi - \varepsilon = \begin{cases} 1-1 & pq \\ 1-0 & p(1-q) \\ 0-1 & (1-p)q \\ 0-0 & (1-p)(1-q) \end{cases} = \begin{cases} 0 & pq+1+pq-p-q \\ 1 & p(1-q) \\ -1 & q(1-p) \end{cases}$$



2.2 B

$$e^{\rho} = \begin{cases} e^1 & p \\ e^0 & 1-p \end{cases} = \begin{cases} e & p \\ 1 & 1-p \end{cases}$$





(2.3) FIND THE MOMENTS (1,2,3) FOR A DIE AND CHECK THAT $VAR = m_2 - m_1^2$

A DIE ξ HAS VALUES 1,2,3,4,5,6 WITH PROBABILITY $\frac{1}{6}$

$$m_1 = E(\xi) = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6} =$$

$$= \frac{1}{6}(1+2+3+4+5+6) = \frac{21}{6} = 3,5$$

$$m_2 = E(\xi^2) = 1^2 \frac{1}{6} + 2^2 \frac{1}{6} + 3^2 \frac{1}{6} + 4^2 \frac{1}{6} + 5^2 \frac{1}{6} + 6^2 \frac{1}{6} = \frac{1}{6}(1+4+9+16+25+36) =$$

$$= \frac{91}{6}$$

$$m_3 = E(\xi^3) = \frac{1}{6}(1+8+27+64+125+216) = \frac{441}{6} = \frac{147}{2}$$

$$VAR(\xi) = \frac{1}{6}\left(1-\frac{21}{6}\right)^2 + \frac{1}{6}\left(2-\frac{21}{6}\right)^2 + \dots + \frac{1}{6}\left(6-\frac{21}{6}\right)^2 =$$

$$= \frac{(6-21)^2 + (12-21)^2 + (18-21)^2 + (24-21)^2 + (30-21)^2 + (36-21)^2}{6 \cdot 36} =$$

$$= \frac{15^2 + 9^2 + 3^2 + 3^2 + 9^2 + 15^2}{216} = \frac{225+81+9+9+81+225}{216} =$$

$$= \frac{630}{216} = \frac{315}{108} = \frac{105}{36} = \frac{35}{12} = 2,91\overline{6}$$

$$m_2 - m_1^2 = \frac{91}{6} - \frac{21^2}{36} = \frac{546-441}{36} = \frac{105}{36} = \frac{35}{12}$$

D

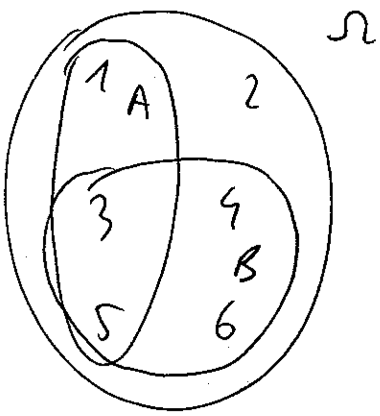
2.4 A

$\mathcal{X}_A(\omega)$ IS INDICATOR ω IS A DIE ROLL

A IS THE SET OF ODD NUMBERS

$\mathcal{X}_B(\omega)$ B IS THE SET OF NUMBERS > 2

FIND $\mathrm{CORR}(\mathcal{X}_A, \mathcal{X}_B)$, $\overset{\text{HW}}{E(\mathcal{X}_A \cdot \mathcal{X}_B)}$, $E(\overset{\text{HW}}{\max}(\mathcal{X}_A, \mathcal{X}_B))$



$$\mathcal{X}_A = \begin{cases} 1 & \text{PROB} = \frac{1}{6}+\frac{1}{6}+\frac{1}{6} = \frac{1}{2} \\ 0 & \text{PROB} = \frac{1}{2} \end{cases}$$

$$\mathcal{X}_B = \begin{cases} 1 & \text{PROB} = \frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6} = \frac{2}{3} \\ 0 & \text{PROB} = \frac{1}{3} \end{cases}$$

$E(\mathcal{X}_A) = \frac{1}{2}$ $E(\mathcal{X}_B) = \frac{2}{3}$ $\mathrm{VAR}(\mathcal{X}_A) = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$ $\mathrm{VAR}(\mathcal{X}_B) = \frac{2}{3}\cdot\frac{1}{3} = \frac{2}{9}$

$$\mathrm{CORR}(\mathcal{X}_A, \mathcal{X}_B) = \frac{\mathrm{COV}(\mathcal{X}_A, \mathcal{X}_B)}{\sqrt{\mathrm{VAR}(\mathcal{X}_A)\cdot\mathrm{VAR}(\mathcal{X}_B)}}$$

TO CALCULATE THE COVARIANCE WE HAVE TWO POSSIBLE WAYS:

1) CALCULATE IT USING THE OUTCOMES OF Ω AND THEIR PROBABILITIES

2) CALCULATE IT USING JOINT PROBABILITY DISTRIBUTION.

<u>WAY 1</u>: $P(1) = \frac{1}{6}$ $P(2) = \frac{1}{6}$ $P(3) = \frac{1}{6}$ $P(4) = \frac{1}{6}$ $P(5) = \frac{1}{6}$ $P(6) = \frac{1}{6}$

$$\mathrm{COV}(\mathcal{X}_A, \mathcal{X}_B) = \sum_{j=1}^{6} \left(\mathcal{X}_A(x_j) - E(\mathcal{X}_A)\right)\cdot\left(\mathcal{X}_B(x_j) - E(\mathcal{X}_B)\right)\cdot P(x_j) =$$

2.4 B

$$= \left(1-\frac{1}{2}\right)\left(0-\frac{2}{3}\right)\cdot\frac{1}{6} + \left(0-\frac{1}{2}\right)\cdot\left(0-\frac{2}{3}\right)\cdot\frac{1}{6} + \left(1-\frac{1}{2}\right)\left(1-\frac{2}{3}\right)\cdot\frac{1}{6} +$$

$$+ \left(0-\frac{1}{2}\right)\left(1-\frac{2}{3}\right)\frac{1}{6} + \left(1-\frac{1}{2}\right)\left(1-\frac{2}{3}\right)\cdot\frac{1}{6} + \left(0-\frac{1}{2}\right)\left(1-\frac{2}{3}\right)\cdot\frac{1}{6} =$$

$$= \frac{1}{6}\left(-\frac{1}{3}+\frac{1}{3}+\frac{1}{6}-\frac{1}{6}+\frac{1}{6}-\frac{1}{6}\right) = 0$$

WAY 2: WE NEED THE JOINT DISTRIBUTION PROBABILITIES

$$P(x_A=0; x_B=0) = P(u=2) = \frac{1}{6}$$

$$P(x_A=1; x_B=0) = P(u=1) = \frac{1}{6}$$

$$P(x_A=0; x_B=1) = P(u=4 \text{ OR } u=6) = \frac{2}{6} = \frac{1}{3}$$

$$P(x_A=1; x_B=1) = P(u=3 \text{ OR } u=5) = \frac{2}{6} = \frac{1}{3}$$

$$\operatorname{COV}(x_A, x_B) = \sum_{\substack{j=1,2\\k=1,2}} \left(x_{A_j} - E(x_A)\right)\cdot\left(x_{B_k} - E(x_B)\right)\cdot P\left(x_{A_j}; x_{B_k}\right) =$$

$$= \left(0-\frac{1}{2}\right)\left(0-\frac{2}{3}\right)\cdot\frac{1}{6} + \left(1-\frac{1}{2}\right)\left(0-\frac{2}{3}\right)\cdot\frac{1}{6} + \left(0-\frac{1}{2}\right)\left(1-\frac{2}{3}\right)\cdot\frac{1}{3} +$$

$$+ \left(1-\frac{1}{2}\right)\left(1-\frac{2}{3}\right)\cdot\frac{1}{3} =$$

$$= \frac{1}{3}\cdot\frac{1}{6} - \frac{1}{18} - \frac{1}{6}\cdot\frac{1}{3} + \frac{1}{2}\cdot\frac{1}{9} = 0$$

$\operatorname{CORR}(x_A, x_B) = 0$ THIS IS NOT ENOUGH TO SAY THEY ARE INDEPENDENT!

IN ORDER TO CHECK INDEPENDENCE: $P(x_A = x; x_B = y) = P(x_A = x)\cdot P(x_B = y)$ $\forall x, y$

2.4 C

$$X_A \cdot X_B = \begin{cases} \omega=1 & 0 \\ \omega=2 & 0 \\ \omega=3 & 1 \\ \omega=4 & 0 \\ \omega=5 & 1 \\ \omega=6 & 0 \end{cases} \qquad X_A \cdot X_B = \begin{cases} 0 & \text{PROB} = \frac{2}{3} \\ 1 & \text{PROB} = \frac{1}{3} \end{cases}$$

$E(X_A \cdot X_B) = \frac{1}{3}$ SINCE IT IS A BERNOULLI.

$$\max(X_A, X_B) = \begin{cases} \omega=1 & 1 \\ \omega=2 & 0 \\ \omega=3 & 1 \\ \omega=4 & 1 \\ \omega=5 & 1 \\ \omega=6 & 1 \end{cases} \qquad \max(X_A, X_B) = \begin{cases} 0 & \text{PROB} = \frac{1}{6} \\ 1 & \text{PROB} = \frac{5}{6} \end{cases}$$

$$E(\max(X_A, X_B)) = \frac{5}{6}$$

$\square$

2.5 X AND Y RANDOM VARIABLES

$VAR(X)=1 \quad VAR(Y)=1$

IS IT POSSIBLE THAT

- $VAR(X+Y) \leq 1$
- $VAR(X+Y)=1,5$?

$$VAR(X+Y) = VAR(X) + VAR(Y) + 2\,COV(X,Y) =$$
$$= 2 + 2\,COV(X,Y)$$

IF $COV(X,Y) < -\frac{1}{2} \Rightarrow VAR(X+Y) < 1$

IF $COV(X,Y) = -\frac{1}{4} \Rightarrow VAR(X+Y) = 1,5$

THESE RESULTS ARE POSSIBLE SINCE

$$COV(X;Y) = \sum_i \left(X(\omega_i) - E(X)\right)\left(Y(\omega_i) - E(Y)\right) P(\omega_i)$$

CAN BE NEGATIVE

D

(2.6)

χ_A INDICATOR $\quad \chi_B$ INDICATOR

EXPRESS $\chi_{A\cup B}$ $\quad \chi_{A\cap B}$ VIA χ_A AND χ_B

$$\chi_{A\cup B} = \begin{cases} 1 & \omega\in A \text{ OR } \omega\in B \\ 0 & \omega\notin A \text{ AND } \omega\notin B \end{cases}$$

$$\chi_A\cdot\chi_B \begin{cases} 1 & \omega\in A \text{ AND } \omega\in B \\ 0 & \omega\notin A \text{ OR } \omega\notin B \end{cases}$$

OK, IT IS GOOD FOR

$$\chi_{A\cap B} = \begin{cases} 1 & \omega\in A \text{ AND } \omega\in B \\ 0 & \omega\notin A \text{ OR } \omega\notin B \end{cases}$$

$$\chi_A+\chi_B = \begin{cases} 2 & \omega\in A \ \ \omega\in B \\ 1 & \omega\in A \ \ \omega\notin B \\ & \omega\notin A \ \ \omega\in B \\ 0 & \omega\notin A \ \ \omega\notin B \end{cases}$$

NO

$$\max(\chi_A,\chi_B) = \begin{cases} 1 & \omega\in A \text{ OR } \omega\in B \\ 0 & \omega\notin A \text{ AND } \omega\notin B \end{cases}$$

OK IT IS $\chi_{A\cup B}$

$$\min(\chi_A,\chi_B) = \begin{cases} 1 & \omega\in A \text{ AND } \omega\in B \\ 0 & \omega\notin A \text{ OR } \omega\notin B \end{cases}$$

$\Rightarrow$ IT IS EXACTLY $\chi_A\cdot\chi_B$

$\square$

(27)

FAMILY WITH 4 CHILDREN

$A = \{$ AT LEAST 1 MALE $\}$

HW $B = \{$ EXACTLY 2 MALES $\}$

HW $C = \{$ 1 OR 2 FEMALE $\}$

HW $D = \{$ NO FEMALES $\}$ CHARACTERIZE X_A, X_B, X_C, X_D

AND FIND: $E(X_A), E(X_B), E(X_C), E(X_D)$

IF WE CONSIDER MALE = 1 AND FEMALE = 0,
IT IS A BINOMIAL DISTRIBUTION WITH $n=4$ $p=0{,}5$

$$P(A) = \binom{4}{1} 0{,}5^1 \cdot 0{,}5^3 + \binom{4}{2} 0{,}5^2 0{,}5^2 + \binom{4}{3} 0{,}5^3\, 0{,}5 + \binom{4}{4} 0{,}5^4 \cdot 0{,}5^0 =$$

$$= \frac{4!}{1!(4-1)!} \frac{1}{2} \cdot \frac{1}{8} + \frac{4!}{2!(4-2)!} \frac{1}{4} \cdot \frac{1}{4} + \frac{4!}{3!\,(4-3)!} \frac{1}{8} \cdot \frac{1}{2} + \frac{4!}{4!(4-4)!} \frac{1}{16} =$$

$$= \frac{4}{16} + \frac{6}{16} + \frac{4}{16} + \frac{1}{16} = \frac{15}{16}$$

$$X_A = \begin{cases} 1 & \frac{15}{16} \\ 0 & \frac{1}{16} \end{cases}$$

$$E(X_A) = 1 \cdot P(A) + 0 \cdot P(CA) = \frac{15}{16}$$

$$P(B) = \binom{4}{2} \frac{1}{2^2} + \frac{1}{2^2} = \frac{6}{16} = \frac{3}{8} \qquad E(X_B) = \frac{3}{8}$$

$$X_B = \begin{cases} 1 & 3/8 \\ 0 & 5/8 \end{cases}$$

$$P(C) = \binom{4}{2} \frac{1}{2} \cdot \frac{1}{2^3} + \binom{4}{2} \frac{1}{2^2} \frac{1}{2^2} = \frac{4}{16} + \frac{6}{16} = \frac{10}{16} = \frac{5}{8} \qquad E(X_C) = \frac{5}{8}$$

$$P(D) = \binom{4}{0} = \frac{4!}{0!(4-0)!} \frac{1}{2^0} \cdot \frac{1}{2^4} = \frac{1}{16} \qquad E(X_D) = \frac{1}{16}$$

$\square$

2.8

A = RESULT OF A DIE ROLL

B = RESULT OF A COIN FLIP WITH 1 IF HEAD, 0 IF TAIL

DESCRIBE JOINT PROBABILITY DISTRIBUTION OF A AND B AND FIND MARGINAL DISTRIBUTION

$$A = \{1, 2, 3, 4, 5, 6\} \quad \text{with probabilities } \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}$$

$$B = \{1, 0\} \quad \text{with probabilities } \tfrac{1}{2}, \tfrac{1}{2}$$

$$C = \left\{ \begin{matrix} (1,1) & (2,1) & (3,1) & (4,1) & (5,1) & (6,1) \\ (1,0) & (2,0) & (3,0) & (4,0) & (5,0) & (6,0) \end{matrix} \right\}$$

PROBABILITIES OF C ARE $\frac{1}{12}$ FOR EVERY RESULT

$P(x,y) = \frac{1}{12} \quad \forall x \, \forall y$ is THE JOINT PROBABILITY DISTRIBUTION

IT IS OBTAINED AS MULTIPLICATION OF PROBABILITIES BECAUSE A AND B ARE INDEPENDENT

$P(x) = \frac{1}{6} \quad \forall x = 1, 2, \ldots 6$ is MARGINAL DISTRIBUTION FOR A

$P(y) = \frac{1}{2} \quad \forall y = 1, 0$ is MARGINAL DISTRIBUTION FOR B

2.9)

A = DIE ROLL RESULT

B = INDICATOR WHETHER DIE ROLL IS 6 (THE SAME DIE ROLL)

FIND JOINT PROBABILITY DISTRIBUTION AND MARGINAL DISTRIB.

$$A = \begin{cases} 1 & 1/6 \\ 2 & 1/6 \\ 3 & 1/6 \\ 4 & 1/6 \\ 5 & 1/6 \\ 6 & 1/6 \end{cases}$$

↓

MARGINAL DISTRIBUTION FOR A

$$B = \begin{cases} \left.\begin{array}{l} 1 \to 0 \\ 2 \to 0 \\ 3 \to 0 \\ 4 \to 0 \\ 5 \to 0 \end{array}\right\} 5/6 \\ \left.6 \to 1 \right\} 1/6 \end{cases}$$

↓

MARGINAL DISTRIBUTION FOR B

JOINT DISTRIBUTION:

1,0	1/6	3,1	NEVER
2,0	1/6	4,1	NEVER
3,0	1/6	5,1	NEVER
4,0	1/6	6,1	1/6
5,0	1/6		
6,0	NEVER		
1,1	NEVER		
2,1	NEVER		

↓

JOINT PROBABILITY DISTRIBUTION

2.10 FIND CORRELATION BETWEEN NUMBER OF DOTS ON A DIE AND χ_A WHERE $A = \{1; 3; 5\}$

SUGGESTION → USE OUTCOMES TO CALCULATE COVAR

$$X = \begin{cases} 1 & 1/6 \\ 2 & 1/6 \\ 3 & 1/6 \\ 4 & 1/6 \\ 5 & 1/6 \\ 6 & 1/6 \end{cases} \qquad \chi_A = \begin{cases} 1 & 1;3;5 \\ 0 & 2;4;6 \end{cases}$$

$$E(X) = \frac{7}{2} \qquad E(\chi_A) = \frac{1}{2}$$

$$\mathrm{COV}(X, \chi_A) = \left(1-\frac{7}{2}\right)\left(1-\frac{1}{2}\right)\frac{1}{6} + \left(2-\frac{7}{2}\right)\left(0-\frac{1}{2}\right)\frac{1}{6} + \left(3-\frac{7}{2}\right)\left(1-\frac{1}{2}\right)\frac{1}{6} +$$

$$+ \left(4-\frac{7}{2}\right)\left(0-\frac{1}{2}\right)\frac{1}{6} + \left(5-\frac{7}{2}\right)\left(1-\frac{1}{2}\right)\frac{1}{6} + \left(6-\frac{7}{2}\right)\left(0-\frac{1}{2}\right)\frac{1}{6} =$$

$$= -\frac{5}{2}\,\frac{1}{2}\,\frac{1}{6} + \frac{3}{2}\,\frac{1}{2}\,\frac{1}{6} - \frac{1}{2}\,\frac{1}{2}\,\frac{1}{6} - \frac{1}{2}\,\frac{1}{2}\,\frac{1}{6} + \frac{3}{2}\,\frac{1}{2}\,\frac{1}{6} - \frac{5}{2}\,\frac{1}{2}\,\frac{1}{6} =$$

$$= \frac{\cancel{-5}+3-1-1+3\cancel{-5}}{24} = -\frac{6}{24} = -\frac{1}{4}$$

$$\mathrm{VAR}(X) = \left(1-\frac{7}{2}\right)^2\frac{1}{6} + \left(2-\frac{7}{2}\right)^2\frac{1}{6} + \left(3-\frac{7}{2}\right)^2\frac{1}{6} + \left(4-\frac{7}{2}\right)^2\frac{1}{6} + \left(5-\frac{7}{2}\right)^2\frac{1}{6} +$$

$$+ \left(6-\frac{7}{2}\right)^2\frac{1}{6} = \frac{1}{24}\left[25+9+1+1+9+25\right] = \frac{70}{24} = \frac{35}{12}$$

$$\mathrm{VAR}(\chi_A) = \left(1-\frac{1}{2}\right)^2\frac{1}{2} + \left(0-\frac{1}{2}\right)^2\frac{1}{2} = \frac{1}{4}\cdot\frac{1}{2} + \frac{1}{4}\cdot\frac{1}{2} = \frac{1}{4}$$

$$\mathrm{CORR}(X, \chi_A) = -\frac{1}{4} \Big/ \sqrt{\frac{35}{12}\cdot\frac{1}{4}} = -\frac{1}{4}\cdot\sqrt{\frac{4\cdot 12}{35}} \approx -0{,}293$$

□

2.11 A

THE JOINT DISTRIBUTION OF X AND Y IS

$P(0;0) = 0.1 \quad P(1;0) = 0,2$

$P(0;1) = 0.2 \quad P(1;1) = 0,1$

$P(0;2) = 0.1 \quad P(1;2) = 0,3$

A) FIND MARGINAL DISTRIBUTIONS OF X AND Y

B) ARE X AND Y INDEPENDENT ?

C) FIND $COV(X,Y)$ HW: USE JOINT PROB.

$$P(X=0) = P(X=0;\ Y=0) + P(X=0;\ Y=1) + P(X=0;\ Y=2) =$$
$$= 0,1 + 0,2 + 0,1 = 0,4$$

$$P(X=1) = P(X=1;\ Y=0) + P(X=1;\ Y=1) + P(X=1;\ Y=2) =$$
$$= 0,2 + 0,1 + 0,3 = 0,6$$

$$P(Y=0) = P(X=0;\ Y=0) + P(X=1;\ Y=0) = 0,1 + 0,2 = 0,3$$

$$P(Y=1) = 0,2 + 0,1 = 0,3$$

$$P(Y=2) = 0,1 + 0,3 = 0,4$$

IN ORDER TO HAVE INDEPENDENCE, WE MUST HAVE

$$P(X=x;\ Y=y) = P(X=x) \cdot P(Y=y) \quad \forall x \forall y$$

$$P(0;0) \stackrel{?}{=} P(X=0) \cdot P(Y=0)$$

$$0,1 \stackrel{?}{=} 0,4 \cdot 0,3$$

$$0,1 \stackrel{!}{=} 0,12$$ <u>NO</u> THEY ARE NOT INDEPENDENT

2.11 B)

$$\mathrm{cov}(X,Y) = \sum_{i,j} (x_i - E(X))(y_j - E(Y)) \cdot p(x_i; y_j)$$

$$E(X) = 0 \cdot 0{,}4 + 1 \cdot 0{,}6 = 0{,}6$$

$$E(Y) = 0 \cdot 0{,}3 + 1 \cdot 0{,}3 + 2 \cdot 0{,}4 = 1{,}1$$

$$\begin{aligned}\mathrm{cov}(X,Y) &= (0-0{,}6)(0-1{,}1)\cdot 0{,}1 + (0-0{,}6)(1-1{,}1)\cdot 0{,}2 + \\ &+ (0-0{,}6)(2-1{,}1)\cdot 0{,}1 + (1-0{,}6)(0-1{,}1)\cdot 0{,}2 + \\ &+ (1-0{,}6)\cdot(1-1{,}1)\cdot 0{,}1 + (1-0{,}6)(2-1{,}1)\cdot 0{,}3 = \\ &= 0{,}066 + 0{,}012 - 0{,}054 - 0{,}088 - 0{,}004 + 0{,}108 = \\ &= 0{,}040\end{aligned}$$

2.12 IT IS BELIEVED THAT EARTHQUAKES IN BOLZANO OCCUR EVERY 6 YEARS.

LAST ONE WAS IN 2008. WHAT IS THE PROBABILITY TO HAVE NONE BEFORE DECEMBER 2017 ?
AND 1? AND 2? AND 3?

RARE EVENTS ARE DISTRIBUTED ACCORDING TO POISSON DISTRIBUTION

$$P(x) = \frac{\lambda^x}{x!} e^{-\lambda} \qquad E(\xi) = \lambda$$

USING NINE YEARS , $\lambda = 1{,}5$

$$P(0) = \frac{1{,}5^0}{0!} e^{-1{,}5} = e^{-1{,}5} \approx 0{,}224 = 22{,}4\%$$

$$P(1) = \frac{1{,}5^1}{1!} e^{-1{,}5} \approx 33{,}6\%$$

$$P(2) = \frac{1{,}5^2}{2!} e^{-1{,}5} \approx 1{,}125 \cdot 0{,}224 = 25{,}2\%$$

$$P(3) = \frac{1{,}5^3}{3!} e^{-1{,}5} \approx 0{,}5625 \cdot 0{,}224 = 12{,}6\%$$