4.1 FIND THE MEDIAN FOR

$$\exp(\alpha N(\mu, \sigma^2)) \qquad \alpha \in \mathbb{R}$$

IT IS ALMOST A LOGNORMAL, EXCEPT FOR A PARAMETER α
WE MAKE THE SAME CALCULATIONS OF THE LOGNORMAL

$$P(\exp(\alpha N) < y) = \int_0^y f_{\exp(\alpha N)}(s)\, ds$$

$$\|$$

$$P(\alpha N < \ln y) = P\left(N < \frac{1}{\alpha} \ln y\right) = \int_{-\infty}^{\frac{\ln y}{\alpha}} f_N(s)\, ds =$$

$$= \int_{-\infty}^{\mu} f_N(s)\, ds + \int_{\mu}^{\frac{\ln y}{\alpha}} f_N(s)\, ds = \frac{1}{2} + \int_{\mu}^{\frac{\ln y}{\alpha}} f_N(s)\, ds$$

WHEN $\frac{\ln y}{\alpha} = \mu$ WE HAVE $\int_0^y f_{\exp(\alpha N)}(s)\, ds = \frac{1}{2}$

$$\Downarrow$$

$$\ln y = \mu\alpha \Rightarrow y = e^{\alpha\mu}$$

OR AN <u>ALTERNATIVE</u> WAY TO PROVE THIS:

WE KNOW THAT $\exp(N(\mu, \sigma^2))$ IS LOGNORMAL AND WE KNOW THAT MEDIAN OF THE LOGNORMAL IS e^{μ}

WE ALSO KNOW THAT $\alpha N(\mu; \sigma^2)$ IS <u>ALSO</u> A NORMAL $N(\alpha\mu; \alpha^2\sigma^2)$

THEREFORE MEDIAN FOR $\exp(N(\alpha\mu; \alpha^2\sigma^2))$ IS $e^{\alpha\mu}$

$\square$

4.2

$\xi \sim U(0,1)$ FIND THE DENSITY f_ζ OF $\zeta = \exp(\xi)$

SUGGESTION: USE SAME TRICK AS FOR LN

$$P(\zeta < y) = \int_0^y f_\zeta(s)\,ds$$

$$\parallel$$

$$P(\exp(\xi) < y) = P(\xi < \ln y) = \int_0^{\ln y} 1 \cdot ds$$

IT IS THE FIRST EXTREME OF $U(0,1)$

IT IS THE DENSITY OF $U(0,1)$

$$= \left[s \right]_0^{\ln y} = \ln y$$

$$\int_0^y f_\zeta(s)\,ds = \ln y$$ WE DERIVE $\frac{d}{dy}$

$$f_\zeta(y) = \frac{1}{y}$$ THIS IS THE DENSITY f_ζ OF ζ

DEFINED ON $(0, +\infty)$