5.1

WE ROLL A DIE 20 TIMES AND WE EVALUATE THE AVERAGE.

WHAT IS $P(\text{AVERAGE}_{20} > 3,7)$?

$$\text{AVERAGE}_{20} = \begin{cases} 1,1,\dots,1,1 & AV=1 & PROB=\left(\frac{1}{6}\right)^{20} \\ 1,1,\dots,1,2 & AV=1,05 & PROB=\left(\frac{1}{6}\right)^{20} \\ 1,1,\dots,1,3 & AV=1,10 & PROB=\left(\frac{1}{6}\right)^{20} \\ \vdots & & \\ 6,6,\dots,6,6 & AV=6 & PROB=\left(\frac{1}{6}\right)^{20} \end{cases}$$

CALCULATING ALL THESE CASES IS VERY LONG, THERE ARE 3656 MILLIONS OF MILLIONS CASES.

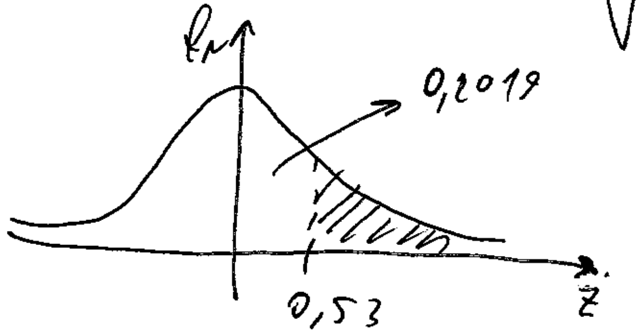
$\Downarrow$

WE APPROXIMATE IT WITH A NORMAL

$$\frac{\bar{X}_{20} - E(x)}{\sqrt{\frac{VAR(x)}{n}}} \approx N(0;1) \quad \text{OR} \quad \bar{X}_{20} \approx N\left(E(x); \frac{VAR(x)}{n}\right)$$

$E(x) = 3,5$ $\quad VAR(x) = (1-3,5)^2\frac{1}{6} + (2-3,5)^2\frac{1}{6} + \dots + (6-3,5)^2\frac{1}{6} =$
$= 2,92$

WHEN $\bar{X}_{20} > 3,7 \Rightarrow \dfrac{\bar{X}_{20} - 3,5}{\sqrt{\frac{2,92}{20}}}$ IS LARGER THAN $\dfrac{3,7-3,5}{0,38} = 0,53$



$P(\bar{X}_{20} > 3,7) = P(z > 0,53) = 0,2981$

D

5.2

CONSIDER A BINOMIAL WITH $p = 0{,}3$

FIND THE MINIMAL n FOR WHICH

$B(0{,}3; n) \approx N$

FROM C.L.T. THE APPROXIMATION IS GOOD AS LONG AS $0 < p \pm 3\sqrt{\frac{p(1-p)}{n}} < 1$

<u>ALL</u> THESE FOUR CONDITIONS MUST BE SATISFIED!

$$\begin{cases} 0 \underset{(1)}{<} 0{,}3 + 3\sqrt{\frac{0{,}3 \cdot 0{,}7}{n}} \underset{(2)}{<} 1 \\ 0 \underset{(3)}{<} 0{,}3 - 3\sqrt{\frac{0{,}3 \cdot 0{,}7}{n}} \underset{(4)}{<} 1 \end{cases} \qquad \begin{cases} 0 \underset{(1)}{<} 0{,}3 + \frac{3\sqrt{0{,}21}}{\sqrt{n}} \underset{(2)}{<} 1 \\ 0 \underset{(3)}{<} 0{,}3 - \frac{3\sqrt{0{,}21}}{\sqrt{n}} \underset{(4)}{<} 1 \end{cases}$$

$$\begin{cases} 0 \underset{(1)}{<} 0{,}3 + \frac{3 \cdot 0{,}459}{\sqrt{n}} \underset{(2)}{<} 1 \\ 0 \underset{(3)}{<} 0{,}3 - \frac{3 \cdot 0{,}459}{\sqrt{n}} \underset{(4)}{<} 1 \end{cases} \qquad \begin{cases} \underset{(1)}{\text{OK}} \quad 3 \cdot 0{,}459 \underset{(2)}{<} 0{,}7\sqrt{n} \\ 3 \cdot 0{,}459 \underset{(3)}{<} 0{,}3\sqrt{n} \quad 3 \cdot 0{,}459 \underset{(4)}{>} -0{,}7\sqrt{n} \end{cases}$$

$$\begin{cases} \underset{(1)}{\text{OK}} \quad \frac{1{,}377}{0{,}7} \underset{(2)}{<} \sqrt{n} \\ \frac{1{,}377}{0{,}3} \underset{(3)}{<} \sqrt{n} \quad \underset{(4)}{\text{OK}} \end{cases} \qquad \begin{cases} \underset{(1)}{\text{OK}} \quad \sqrt{n} \underset{(2)}{>} 1{,}9671 \\ \sqrt{n} \underset{(3)}{>} 4{,}59 \quad \underset{(4)}{\text{OK}} \end{cases} \Rightarrow \boxed{n > 21{,}06}$$

FROM $n = 22$ WE MAY APPROXIMATE $B(0{,}3; n)$ WITH N

D

5.3 A ROLL 2 DICE AND SUM. REPEAT FOR 100 TIMES.

FIND THE PROBABILITY TO OBTAIN 7 MORE THAN 25 TIMES

OUR STOCASTIC EXPERIMENT IS: ROLL, SUM, OBTAIN 7

ROLLING TWO DICE AND SUMMING HAS THE FOLLOWING PROBABILITY:

X	1	2	3	4	5	6	7	8	9	10	11	12
P	/	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

OBTAIN 7 IS AN INDICATOR ⇒ 1 1/6 ; 0 5/6

THIS IS A BERNOULLI EXPERIMENT. WE REPEAT IT 100 TIMES AND EVALUATE THE SUCCESSES

⇓

IT IS A BINOMIAL

$$P(B=Y) = \binom{n}{Y} p^Y (1-p)^{n-Y} = \binom{100}{Y} \left(\frac{1}{6}\right)^Y \left(\frac{5}{6}\right)^{100-Y}$$

TO SOLVE THE PROBLEM WE SHOULD COMPUTE

$P(B=26) + P(B=27) + \dots + P(B=100)$ OR

$1 - P(B=0) + \dots + P(B=25)$

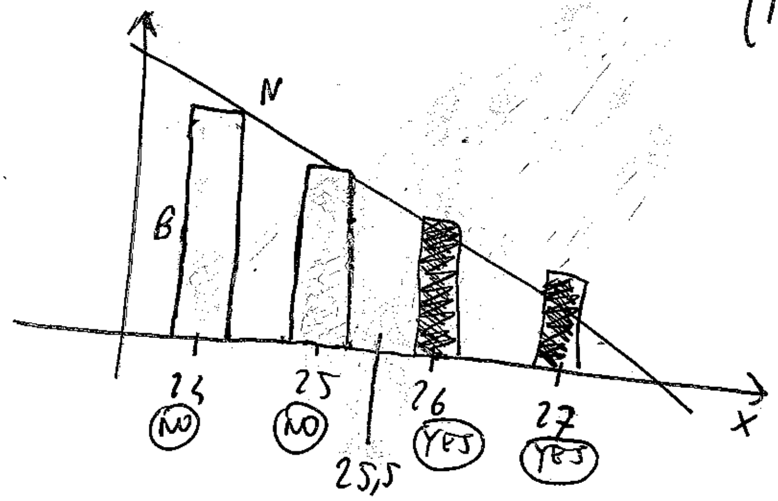
IT IS <u>VERY</u> LONG

(5.3) $\boxed{B}$

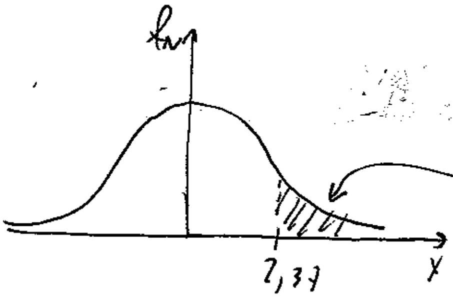
WE MAY APPROXIMATE $B\left(\frac{1}{6}; 100\right)$ WITH A NORMAL DISTRIBUTION USING VALUES

OR
$$z = \frac{B - \frac{100}{6}}{\sqrt{100 \cdot \frac{1}{6} \cdot \frac{5}{6}}} \approx N(0;1)$$

$B > 25$ MUST BE CONVERTED INTO $B > 25,5$ TO TAKE INTO ACCOUNT THAT B IS DISCRETE (IT HAS VALUE 25 AND VALUE 26) BUT z IS CONTINUOUS. FROM B POINT OF VIEW, $B > 25$ MEANS $\{26, 27 \ldots 100\}$ AND $B > 25,5$ MEANS $\{26, 27 \ldots 100\}$ SO IT DOES NOT MAKE DIFFERENCE, BUT FOR z IT MAKES A LITTLE DIFFERENCE (IN THIS CASE, 0,4% OF DIFFERENCE)



$B > 25,5 \Rightarrow z > \left(\frac{51}{2} - \frac{100}{6}\right) \cdot \frac{6}{10\sqrt{5}} = \frac{53}{10\sqrt{5}} = 2,37$



$P(z > 2,37) = 0,0089 = 0,89\%$

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5.4 A MALE UNIVERSITY STUDENTS HAVE THEIR WEIGHT NORMALLY DISTRIBUTED WITH $\mu = 68$ KG AND $\sigma = 3$ KG ($\sigma^2 = 9$ KG^2)

WE TAKE A SAMPLE OF 25 STUDENTS

WHAT IS THE PROBABILITY THAT $\bar{X}_{25} \in [66{,}8; 68{,}3]$?

HW: AND THAT $\bar{X}_{25} < 66{,}4$ KG ?

IF WE ASSUME THAT MALE UNIVERSITY STUDENTS ARE MANY ENOUGH, WE DO NOT CONSIDER THE EVENT OF TAKING THE SAME STUDENT MORE THAN ONE TIME IN THE SAME SAMPLE.

A AVERAGE OF A RANDOM NORMAL VARIABLE IS DISTRIBUTED NORMALLY (ALWAYS, EVEN WHEN n IS SMALL) WITH

$$\mu_{\bar{X}_{25}} = \mu = 68 \text{ KG}$$

$$\sigma_{\bar{X}_{25}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{5} \text{ KG}$$

$\bar{X}_{25} = 66{,}8 \Rightarrow$
$$z = \frac{\bar{X}_{25} - 68}{\frac{\sigma}{\sqrt{n}}} = \frac{\bar{X}_{25} - 68}{\frac{3}{5}} = \frac{66{,}8 - 68}{\frac{3}{5}} =$$

$$= -1{,}2 \cdot \frac{5}{3} = -2$$

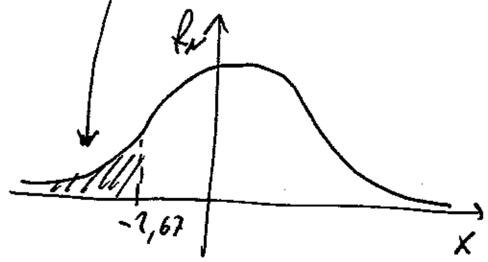
$$\bar{X}_{25} = 68{,}3 \Rightarrow z = \frac{68{,}3 - 68}{\frac{3}{5}} = 0{,}3 \cdot \frac{5}{3} = 0{,}5$$

$$P([-2; 0{,}5]) = P([0; 0{,}5]) + P([0; 2]) = 0{,}1915 + 0{,}4772 = 66{,}87\%$$

5.4 B

$$\overline{X}_{75} = 66,4 \Rightarrow z = \frac{66,4 - 68}{\frac{3}{5}} = -1,6 \cdot \frac{5}{3} = -2,67$$

$$P\left(\left(-\infty; -2,67\right]\right) = \underline{0,0038 = 0,38\%}$$



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