9.0A

PEARSON'S CORRELATION

X	x_1 .... x_n
Y	y_1 .... y_n

VALUES ARE COUPLED

$$SS_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2 \quad \text{OR} \quad SS_{xx} = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$$

$$SS_{xy} = \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) \qquad SS_{xy} = \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}$$

$$\rho = \frac{SS_{xy}}{\sqrt{SS_{xx}\, SS_{yy}}}$$

SPEARMAN'S CORRELATION

IT IS CALCULATED IN THE SAME WAY, BUT RANKS INSTEAD OF VALUES ARE USED.

CASES, COUPLED, MUST BE PUT IN AN ARBITRARY ORDER (ALPHABETICAL, CHRONOLOGICAL, ...)

X	x_1 x_2 .... x_n
Y	y_1 y_2 .... y_n

RANKS →

X	7 12 3
y	9 5 6

$$SS_{xx} = \sum_{i=1}^{n} (r_{ix} - \bar{r})^2 = \sum_{i=1}^{n} r_{ix}^2 - n\bar{r}^2$$

FROM 1 TO n $\quad \bar{r} = \frac{n+1}{2}$

$$SS_{xy} = \sum_{i=1}^{n} (r_{ix} - \bar{r})(r_{iy} - \bar{r}) = \sum_{i=n}^{n} r_{ix} \cdot r_{iy} - n\bar{r}^2$$

$$r_s = \frac{SS_{xy}}{\sqrt{SS_{xx}\, SS_{yy}}} \quad \text{OR} \quad r_s = 1 - \frac{6}{n(n^2-1)} \sum_{i=1}^{n} d_i^2 \qquad d_i = r_{ix} - r_{iy}$$

↓

VALID ONLY IF NO DATUM IS TIED!

9.0 B

HYPOTHESIS TEST

TWO TAILED
$$\begin{cases} H_0: r_s = 0 \\ H_1: r_s \neq 0 \end{cases}$$

ONE TAILED
$$\begin{cases} H_0: r_s \geq 0 \\ H_1: r_s < 0 \end{cases}$$

REJECTION AREA

$$|r_s| > r\left(\frac{\alpha}{2}, n\right)$$

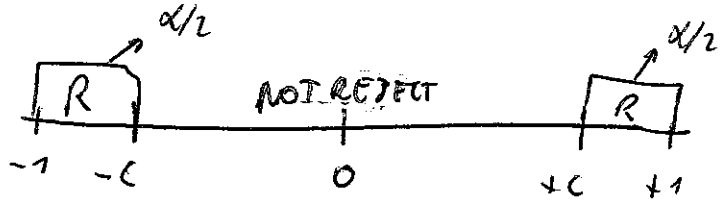
REJECTION AREA

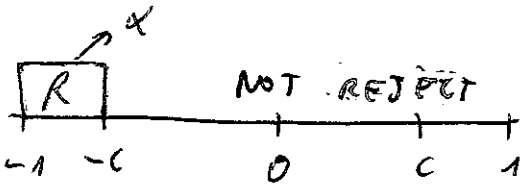
$$r_s < -r(\alpha, n)$$

FOR $\alpha = 0{,}025$

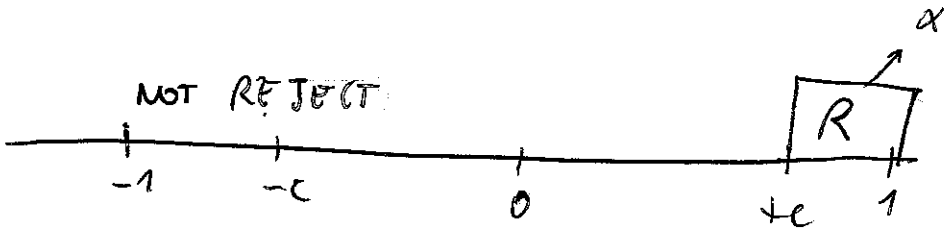
CRITICAL VALUES ARE

n	6	7	8	9	10
$r(\alpha, n)$	0,886	0,786	0,738	0,683	0,648





ONE-TAILED:
$$\begin{cases} H_0: r_s \leq 0 \\ H_1: r_s > 0 \end{cases}$$



9.1 CALCULATE SPEARMAN CORRELATION FOR

X	1,7	3,5	6,2	1,7
Y	2,4	1,6	2,3	6,2

WE CANNOT USE SHORT FORMULA SINCE THERE ARE TIES!

THEREFORE $r_s = \frac{SS_{xy}}{\sqrt{SS_{xx}\, SS_{yy}}}$ WITH RANKS AND NOT VALUES

RANKS:

X	1,5	3	4	1,5
Y	3	1	2	4

$\bar{x} = 2,5$

$\bar{y} = 2,5$

$$SS_{xy} = \sum_{j=1}^{4} x_j y_j - n\,\bar{x}\,\bar{y} = 1,5 \cdot 3 + 3 \cdot 1 + 4 \cdot 2 + 1,5 \cdot 4 - 4 \cdot 2,5 \cdot 2,5 =$$

$$= 4,5 + 3 + 8 + 6 - 25 = 21,5 - 25 = -3,5$$

$$SS_{xx} = \sum_{j=1}^{4} x_j^2 - n\,\bar{x}^2 = (1,5)^2 + 9 + 16 + (1,5)^2 - 4 \cdot (2,5)^2 =$$

$$= 2,25 + 25 + 2,25 - 25 = 4,5$$

$$SS_{yy} = \sum_{j=1}^{4} y_j^2 - n\,\bar{y}^2 = 3^2 + 1^2 + 4 + 16 - 4 \cdot (2,5)^2 = 30 - 25 = 5$$

$$r_s = \frac{-3,5}{\sqrt{4,5 \cdot 5}} = -\frac{3,5}{4,743} = -0,738$$

9.2 TWO CONSUMERS ARE ASKED TO PUT POSSIBLE BICYCLE COLOURS IN ORDER OF PREFERENCE. RESULTS ARE

① RED, YELLOW, BLACK, WHITE, GREEN, BLUE
② BLUE, RED, YELLOW, GREEN, BLACK, WHITE

ARE THEIR PREFERENCES EQUIVALENT?

THE FIRST THING TO DO HERE IS TO PUT COLOURS IN AN ORDER TO COUPLE ① WITH ②. WE MAY CHOOSE, FOR EXAMPLE, ALPHABETICAL ORDER, AND WE ASSIGN THE SCORES:

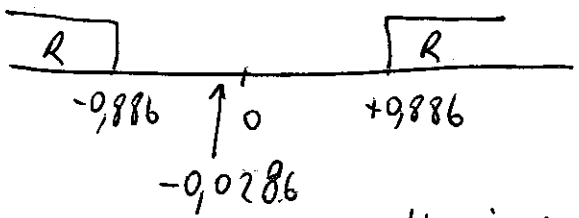
	BLACK	BLUE	GREEN	RED	WHITE	YELLOW
①	3	6	5	1	4	2
②	5	1	4	2	6	3

$r_S = \frac{SS_{12}}{\sqrt{SS_{11}\,SS_{22}}}$ BUT IT IS EASIER $r_S = 1 - \frac{6}{n(n^2-1)} \sum_{j=1}^{n} d_j^2 =$

$= 1 - \frac{6}{6(36-1)} \left(1^2+5^2+1^2+1^2+2^2+1^2\right) = 1 - \frac{36}{35} = -0{,}0286$

$\begin{cases} H_0: \text{RANKS ARE INDEP. } r_S = 0 \\ H_1: \text{RANKS ARE DEPENDENT } r_S \neq 0 \end{cases}$

CRITICAL LEVEL FOR $\alpha = 5\%$ $n=6$ IS $0{,}886$



H_0 IS NOT REJECTED

⇓

WE DO NOT HAVE ENOUGH EVIDENCE TO SAY THAT $r_S \neq 0$

□

9.3

Differences and meanings of some non parametric statistical tests.

Wilcoxon Sum Rank test checks whether two distributions are the same or are different. It is used when you have two sets of data and are simply interested into testing whether they come from the same distribution or not.

Wilcoxon Signed Rank test checks whether two distributions OF PAIRED DATA are different. It can be used only when data are paired and analyzed strongly the difference datum with datum (INTRA-case, inside the same case).

Spearman rank correlation coefficient, and its corresponding test, checks whether two distributions OF PAIRED DATA have the same order or a perfectly reverse order (according to the r_S result). It does not matter whether the data come from the same distribution or not, the important thing is that they are in order. It can be used only when data are paired and when you are simply interested in the order.

PAY ATTENTION that the three tests measure different things, even through whenever your data are paired you can do all of them.

Theoretical examples:

- if data are in perfect reverse order (1,2,3,4,5 and 5,4,3,2,I), Spearman is equal to -1 (H_0 rejected, therefore significant) indicating that the order is reversed while Wilcoxon Sum Rank test and Wilcoxon Signed Rank test do not reject H_0 indicating that data may come from the same distribution.

- if data are perfectly shifted (1,2,3,4,5 and 3,4,5,6,7), Wilcoxon Signed Rank test and Wilcoxon Sum Rank test reject H_0 indicating that data do not come from the same distribution while Spearman is equal to +1 (H_0 rejected, therefore significant) indicating that the order is the same.

Practical examples:

- *given two students and their exams' grades on the same 6 economics subjects, which one will you hire for a position in a bank?*
  Data are paired so we can use all three tests. However, we are not interested whether the two students have the same order or not, but simply whether their two distributions are equivalent or not (and, if we want to do also one-tailed tests, whether one of the two students has exams' grades shifted to the right).
  For example, suppose that grades are 30 29 28 27 26 25 and 25 26 27 28 29 30: in this case for us the two students are equivalent, but Spearman is -1 (H_0 rejected, therefore significant) indicating that the order is reversed, while Wilcoxon tests both do not reject indicating that data may come from the same distribution. Suppose instead that grades are 30 29 28 27 26 25 and 25 24 23 22 21 20: in this case the first student is evidently the best. Spearman is +1 (H_0 rejected, therefore significant) indicating that the order is the same, while Wilcoxon tests both reject indicating that data come from different distributions.
  So the best choice here is Wilcoxon Sum Rank Test, followed by Wilcoxon Signed Rank Test since the differences "exam by exam" are not important. Spearman is not good here since we are not interested in the order.

9.4

- *given two subjects and the grades given to the same 6 students, how can you test whether subject B has marks which have been inflated?*
  Data are paired so we can use all three tests. However, we are not interested whether the two exams have the same order or not (because it can happen that a student is good in a subject but bad in another, due to personal preferences), but we are interested whether they come from the same distribution.
  For example, suppose that grades are 30 29 28 27 26 25 and 25 26 27 28 29 30: in this case, even though the same student got different grades, for each student who has got high grade in A and low in B there is another who compensate with a low grade in A and high in B (and this is not an indication that grades have been inflated, but simply that students good in A are not good in B, due to personal preferences). So subject B has not been inflated. Spearman is -1 (H_0 rejected, therefore significant) indicating that marks are in the reverse order, an information which is totally useless here, while the two Wilcoxon tests do not reject indicating that marks may come from the same distribution. Suppose instead that grades are 25 24 23 22 21 20 and 30 29 28 27 26 25: in this case exam's grades are evidently inflated. Spearman is +1 (H_0 rejected, therefore significant) indicating that marks are in the same order, an useless information, while Wilcoxon tests both reject, indicating that marks do not come from the same distribution.
  So the best choice here is Wilcoxon Sum Rank Test, followed by Wilcoxon Signed Rank Test since the differences "subject by subject" are not important. Spearman is not good here. If, on the other hand, we want to concentrate the attention on the grades inflation subject by subject, Wilcoxon Signed Rank test is the best choice, followed by Wilcoxon Sum Rank text.

- *given two subjects and the grades given to the same 6 students, how can you test whether grades are consistent?*
  Data are paired so we can use all three tests. Consistent here means that good students in one subject are also good in the other. So in this case we are interested in discovering whether grades have the same order or not.
  For example, suppose that grades are 30 29 28 27 26 25 and 25 26 27 28 29 30: in this case it is clear that good students in first subject are bad in the second. Spearman is -1 (H_0 rejected, therefore significant) indicating that the order is different while the two Wilcoxon tests do not reject indicating that marks may come from the same distribution, a useless information in this case. Suppose instead that grades are 25 24 23 22 21 20 and 30 29 28 27 26 25: in this case, even though second exam's grades are evidently inflated, at least good students in first exam are still the best ones in the second. Spearman is +1 (H_0 rejected, therefore significant) indicating that the order is the same, while Wilcoxon tests both reject indicating that data come from different distributions.
  Therefore Spearman is the best choice, and Wilcoxon tests are not appropriate.

- *given two subjects and the grades given to 4 students for subject A and to 8 students (including the previous 4) for subject B, how can you test whether subject B has marks which have been inflated?*
  Data are paired only for the first 4. So using Wilcoxon Signed Rank test implies taking very few subjects and if you look at the table with 4 subjects you see that we must always not reject. Therefore Wilcoxon Sum Rank test with 4/8 subjects is the only test possible here.