11.0 TIME LINEAR REGRESSION

$$Y_j = \beta_0 + \beta_1 t + E_j \quad \text{OR} \quad Y_j = \beta_0 + \beta_1 X_{1j} + \beta_2 X_{2j} + E_j$$

WITH X_1 AND X_2 DUMMY VARIABLES RELATED WITH TIME (DAYS OF WEEK, MONTHS, QUARTERS...)

$\hat{E}_j = Y_j - \hat{Y}_j$ IS THE VALUE OF R.V. E_j (WHICH IS R.V. BECAUSE $E_j = Y_j - \hat{\beta}_0 - \hat{\beta}_1 X_j$ IS COMBINATION OF R.V.)

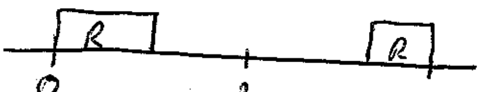
WE KNOW THAT $\sum_{j=1}^{n} \hat{E}_j = \sum_{j=1}^{n} Y_j - \sum_{j=1}^{n} \hat{Y}_j = n\bar{Y} - n\hat{\beta}_0 - n\hat{\beta}_1 \bar{X} =$
$= n\bar{Y} - n(\bar{Y} - \hat{\beta}_1 \bar{X}) - n\hat{\beta}_1 \bar{X} = 0$ THIS MEANS THAT $\hat{E}_j$ ARE <u>NOT</u> INDEPENDENT SINCE $E_1 = -\sum_{j=2}^{n} E_j$

SINCE $\hat{E}_j$ ARE THE VALUES OF E_j WE CAN SUSPECT THAT ALSO E_j ARE DEPENDENT (WHICH PREVENTS US FROM PERFORMING USEFULNESS TEST).
THEREFORE WE BUILD A SPECIFIC TEST, DURBIN-WATSON, WHICH USES $\hat{E}_j$ TO PREDICT CORRELATION OF E_j.

$$\begin{cases} H_0: E_j \text{ NOT AUTOCORRELATED} \\ H_1: E_j \text{ AUTOCORRELATED} \end{cases} \qquad \begin{cases} H_0: E_j \text{ NOT POSITIVELY AUTOCORRELATED} \\ H_1: E_j \text{ POSITIVELY AUTOCORRELATED} \end{cases}$$

STATISTICS IS $d = \dfrac{\sum_{j=2}^{n} (\hat{E}_j - \hat{E}_{j-1})^2}{\sum_{j=1}^{n} \hat{E}_j^2}$

d TAKES VALUES FROM 0 (CORR = +1) TO 4 (CORR = −1). $d = 2$ IS CORR = 0



IF E_j ARE NOT AUTOCORRELATED <u>USUALLY</u> THIS DOES NOT MEAN THAT E_j ARE INDEPENDENT. BUT SINCE $E_j \sim N(0; \sigma^2)$, UN CORR $\Rightarrow$ INDEPENDENT

11.1) A THE SELLING OF CALCULATORS IS

	JAN-MAR	APR-JUN	JUL-SEP	OCT-DEC
1992	438	398	252	160
1993	464	429	376	216
1994	523	496	425	318
1995	593	576	456	398
1996	636	640	526	498

BUILD A TIME MODEL WITH SEASONAL COMPONENTS AND TEST ITS USEFULNESS AND CHECK FOR ERRORS' AUTOCORRELATION $\alpha = 10\%$

$$Y_t = \beta_0 + \beta_1 t + \beta_2 Q_1 + \beta_3 Q_2 + \beta_4 Q_3$$

$t = 1$ FOR JAN-MAR 1992 $t = 2$ FOR APR-JUN 1992 ... ... $t = 20$ FOR OCT-DEC 1996

$$Q_1 = \begin{cases} 1 & t = 1, 5, 9, 13, 17 \\ 0 & \text{OTHER} \end{cases} \qquad Q_2 = \begin{cases} 1 & t = 2, 6, 10, 14, 18 \\ 0 & \end{cases}$$

$$Q_3 = \begin{cases} 1 & t = 3, 7, 11, 15, 19 \\ 0 & \text{OTHER} \end{cases}$$

AFTER ALL CALCULATIONS ..... - - .

$\hat\beta_0 = 119,85$ $\hat\beta_1 = 16,5125$ $\hat\beta_2 = 262,3375$

$\hat\beta_3 = 222,825$ $\hat\beta_4 = 105,5125$

$$d = 0,9692$$

$$\Downarrow$$

$$f = \frac{0,9692}{4} \quad \frac{20-5}{0,0308} = 118 \qquad F_{4,15} = 3,06$$

11.1 B H_0 IS REJECTED $\Rightarrow$ THE MODEL IS USEFUL

TO CHECK FOR AUTOCORRELATION, WE USE DURBIN-WATSON TEST

$$\begin{cases} H_0: \text{RESIDUALS UNCORRELATED} \\ H_1: \text{RESIDUALS FIRST-ORDER CORRELATED} \end{cases}$$

$$\hat{E}_t = Y_t - \hat{Y}_t \qquad d = \frac{\sum_{t=2}^{n} (\hat{E}_t - \hat{E}_{t-1})^2}{\sum_{t=1}^{n} \hat{E}^2} = 1{,}3052$$

$$\alpha = 10\%$$

$$\text{REJECTION} \begin{cases} d < d_{L,\frac{\alpha}{2}} \\ \text{OR } d > 4 - d_{L,\frac{\alpha}{2}} \end{cases}$$

WITH 4 IND. VAR. AND 20 OBS. $d_{L,\frac{\alpha}{2}} \approx 0{,}40$

$\Downarrow$

WE DO NOT REJECT H_0, NO AUTOCORRELATION

$\Downarrow$

SINCE $E_j \sim N(0;\sigma^2)$ NO AUTOCORRELATION IMPLIES INDEPENDENCE

D

(11.2) A THE BUYING POWER OF AMERICAN DOLLAR IS

1981 −0,3321
1982 −0,2947
1983 −0,2123
1984 −0,1439
1985 −0,0645
1986 0,0379
1987 0,1063
1988 0,1717
1989 0,2141
1990 0,2615
1991 0,3329

FIT THE MODEL $Y_t = \beta_0 + \beta_1 t + E_t$

AND CHECK FOR ERRORS' AUTOCORR.

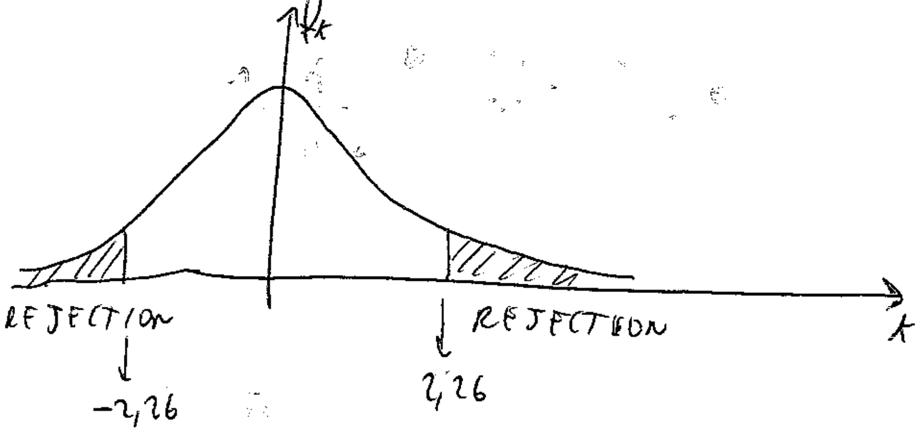
$t = 0$ 1981 $t = 10$ 1991 $\bar{t} = 5$ $\bar{Y}_{11} = 0{,}0079$

$SS_{xx} = 110$ $SS_{xy} = 7{,}591$ $SS_{yy} = 0{,}528169$

$\hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} = 0{,}068$ $\hat{\beta}_0 = \bar{Y}_{11} - \hat{\beta}_1 \bar{t} = -0{,}334$

$r^2 = \frac{SS_{xy}^2}{SS_{xx} SS_{yy}} = 0{,}9940$ $t = \sqrt{n-2}\ SS_{xy} \sqrt{SS_{xx} SS_{yy} - SS_{xy}^2} = 13{,}04$

WITH 9 D.F.



SINCE $t = \frac{\hat{\beta}_1 \sqrt{SS_{xx}}}{\sqrt{SSE}}$

TO ACCEPT H_0 THAT $\hat{\beta}_1 = 0$ WE WANT t SMALL, THEREFORE REJECTION AREA IS ON THE OUTSIDE

H_0 IS REJECTED $\Rightarrow$ THE MODEL IS USEFUL

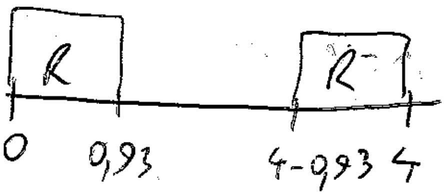
(11.2) B DURBIN-WATSON TEST

H_0: RESID. NON CORRELATED

$$\hat{E}_t = Y_t - \hat{Y}_t \qquad d = \frac{\sum_{t=1}^{10} \left(\hat{E}_t - \hat{E}_{t-1}\right)^2}{\sum_{t=0}^{10} \hat{E}_t^2} = 0,683$$

$\alpha = 5\%$

FOR $N = 11$ $\quad d_{L,0,05} = 0,93$



⇓

H_0 IS REJECTED

⇓

THERE IS AUTOCORRELATION

⇓

REGRESSION MODEL ASSUMPTION DOES NOT HOLD

⇓

WE CANNOT PERFORM USEFULNESS TEST

D